

Dispersive analysis of $\pi\pi$ and πK scattering

Oleksandra Deineka



arXiv: [2012.11636](https://arxiv.org/abs/2012.11636) [hep-ph]

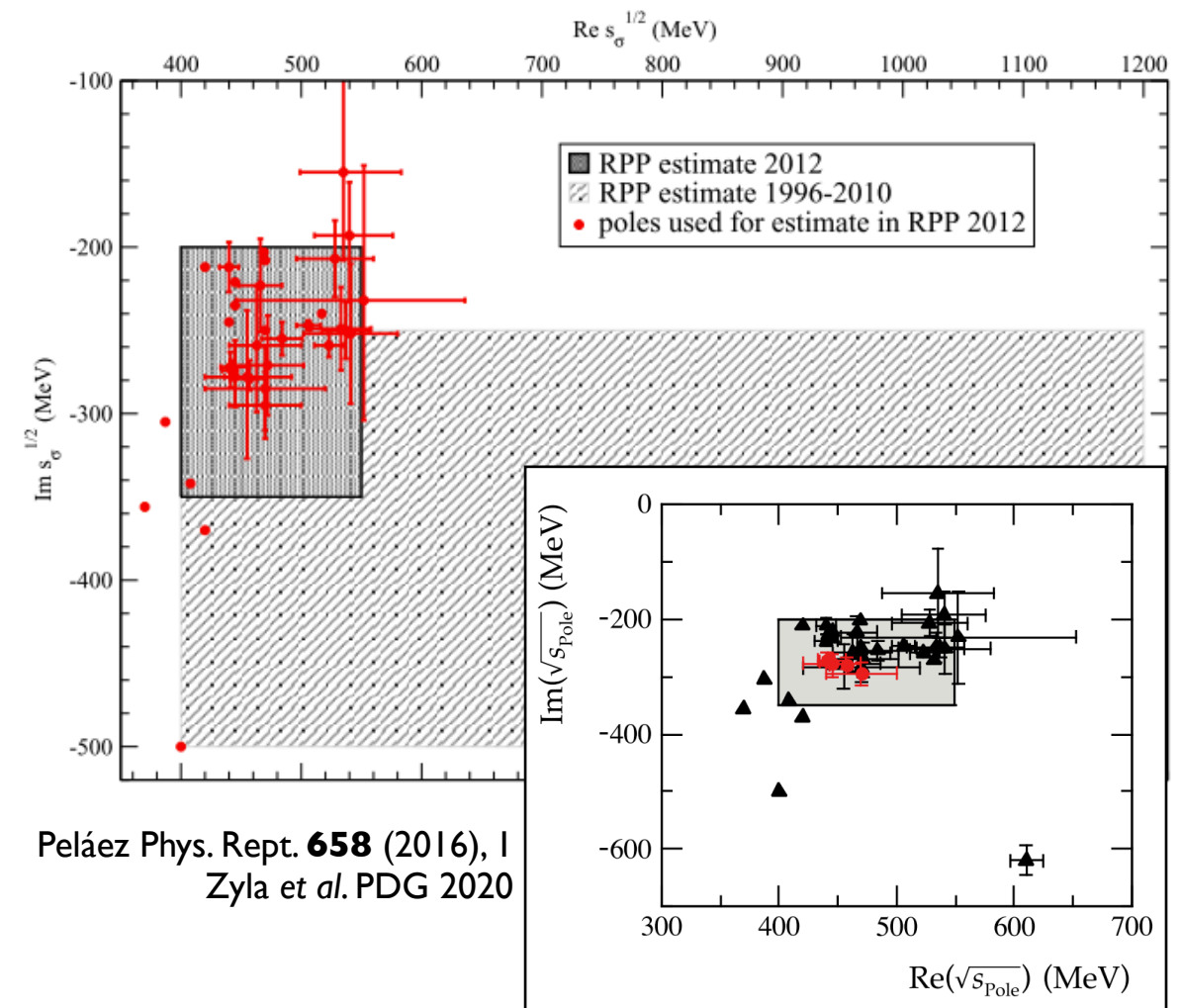
18.05.2021

In collaboration with Igor Danilkin and Marc Vanderhaeghen



Motivation: resonances

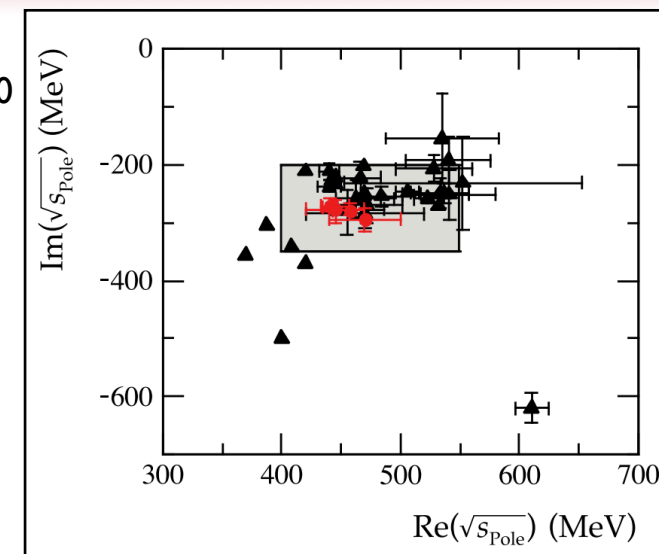
- Scalar resonances: $f_0(500)$, $f_0(980)$, $K^*(700)$
- Precise dispersive determination for $f_0(500)$ and $K^*(700)$ from Roy and Roy-Steiner analyses based on all available **experimental** data



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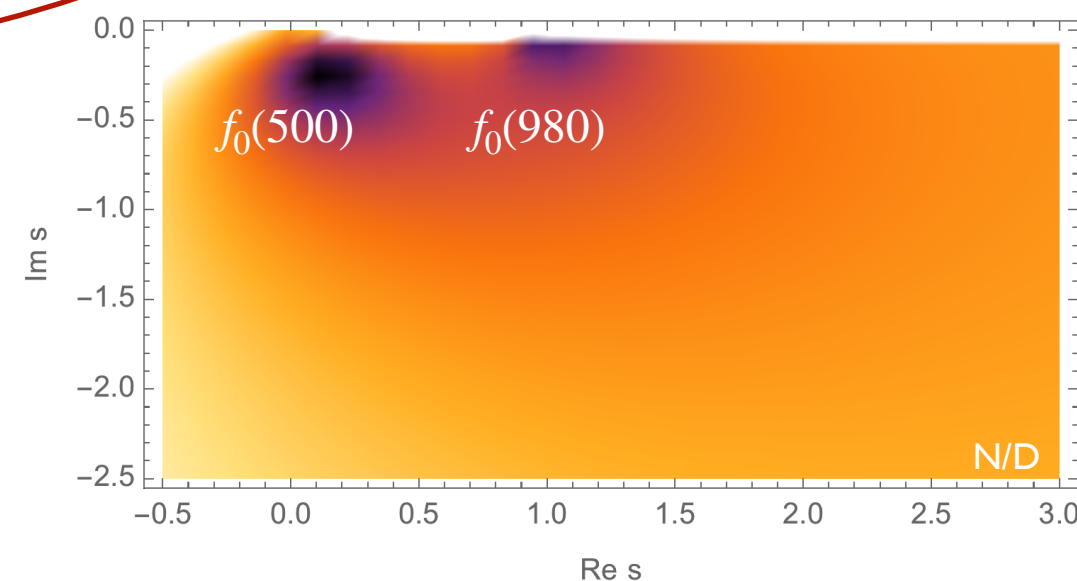
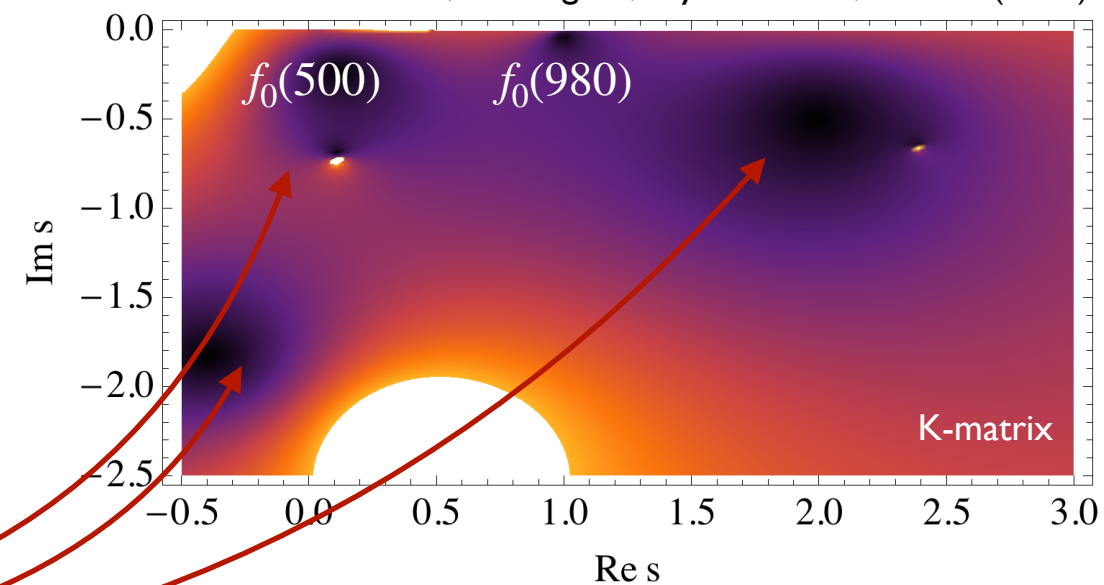
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Zyla et al. PDG 2020



- Recent **lattice** studies of $\pi\pi \rightarrow \pi\pi$ ($\pi K \rightarrow \pi K$) for pion masses as low as 236 (239) MeV
- Lattice data is analysed using K-matrix approach \implies not always possible to find a stable pole due to the presence of spurious poles
- We use N/D approach which incorporates both unitarity and analyticity

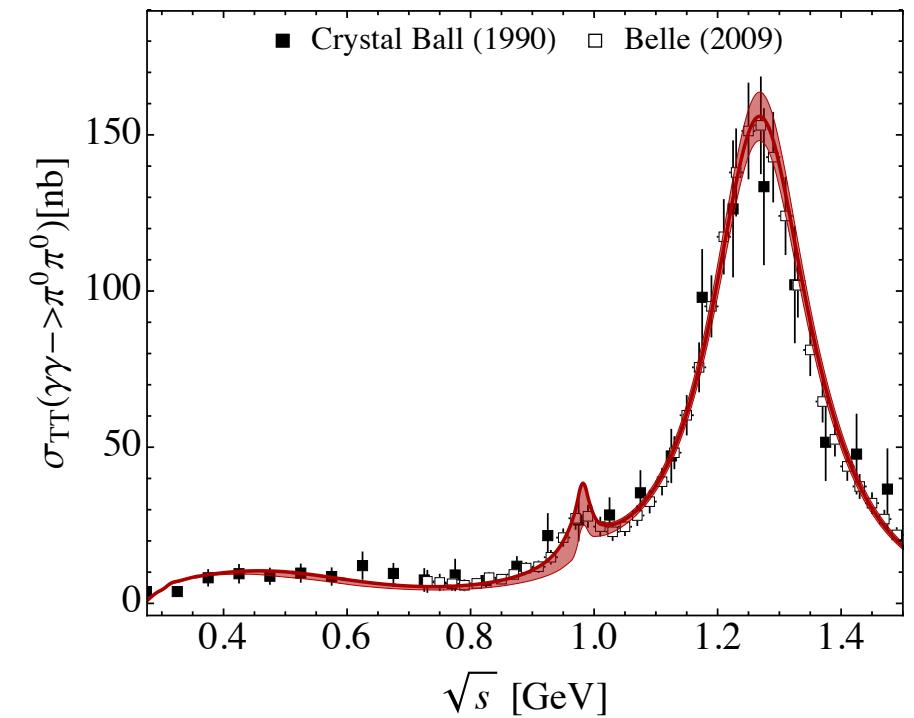
Dai, Pennington, Phys. Rev.D90, 036004 (2014)



Motivation: $\gamma\gamma \rightarrow \pi\pi$ and other reactions input

- The **Omnès function** is crucial and universal: it accounts for the hadronic FSI. It arises naturally from the N/D approach as an inverse of the D -function
- Input to muon ($g - 2$) from single and double virtual $\gamma^*\gamma^* \rightarrow \pi\pi, K\bar{K}, \pi\eta$ processes
- Recent calculation of $f_0(980)$ contribution

$$a_\mu^{\text{HLbL}}[f_0(980)] = -0.2(2) \times 10^{-11} \text{ arXiv: 2105.01666 [hep-ph]}$$



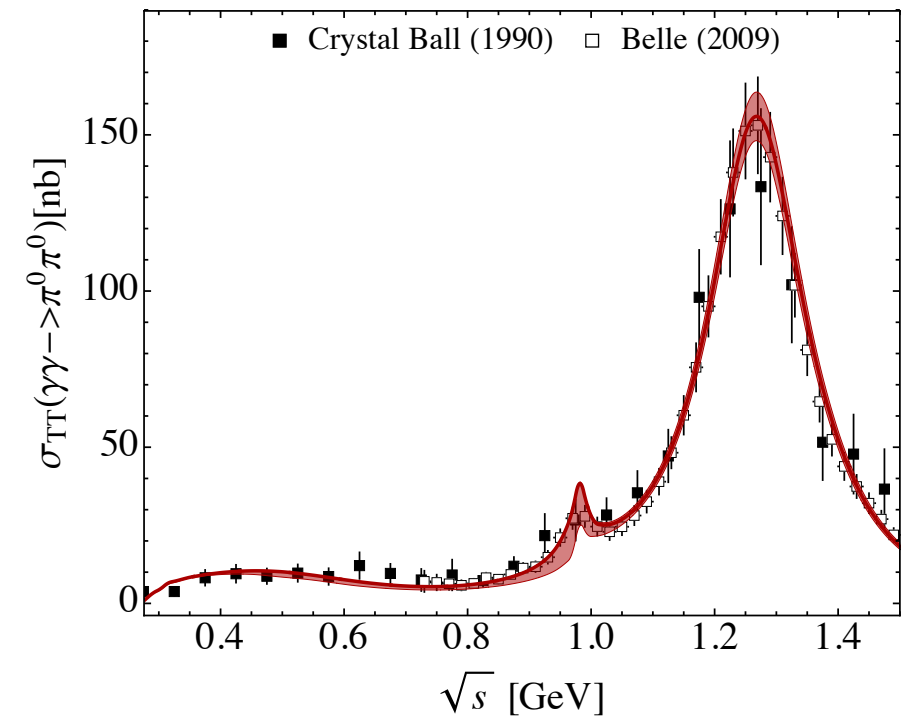
Acta Phys.Polon.B 50 (2019) 1901-1910

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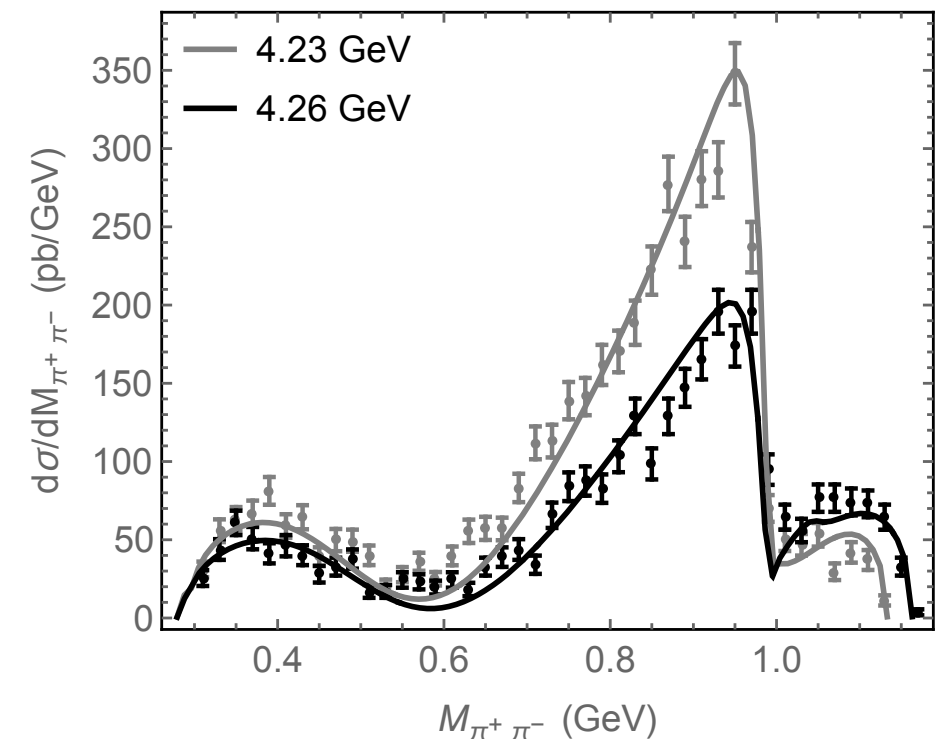
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- Further application to exotics: $Z_c(3900)$ from $e^+e^- \rightarrow J/\psi\pi\pi(K\bar{K})$



Acta Phys.Polon.B 50 (2019) 1901-1910



Phys. Rev. D 102 1, 016019, (2020)

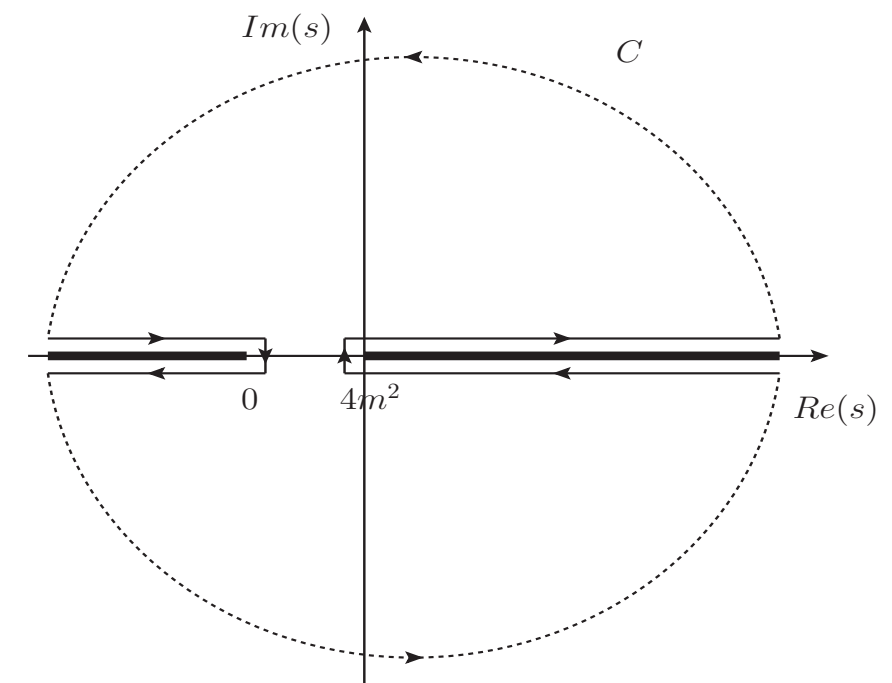
Partial wave dispersion relation

Unitarity relation for the partial wave amplitudes \implies guarantees that p.w. amplitudes behave asymptotically **no worse than a constant**

$$\text{Disc } t_{ab}(s) = \sum_c t_{ac}(s) \rho_c(s) t_{cb}^*(s)$$

From the **maximal analyticity** principle one can write **dispersion relation**

$$t_{ab}(s) = \int_{-\infty}^{s_L} \frac{ds'}{\pi} \frac{\text{Disc } t_{ab}(s')}{s' - s} + \int_{s_{th}}^{\infty} \frac{ds'}{\pi} \frac{\text{Disc } t_{ab}(s')}{s' - s}$$



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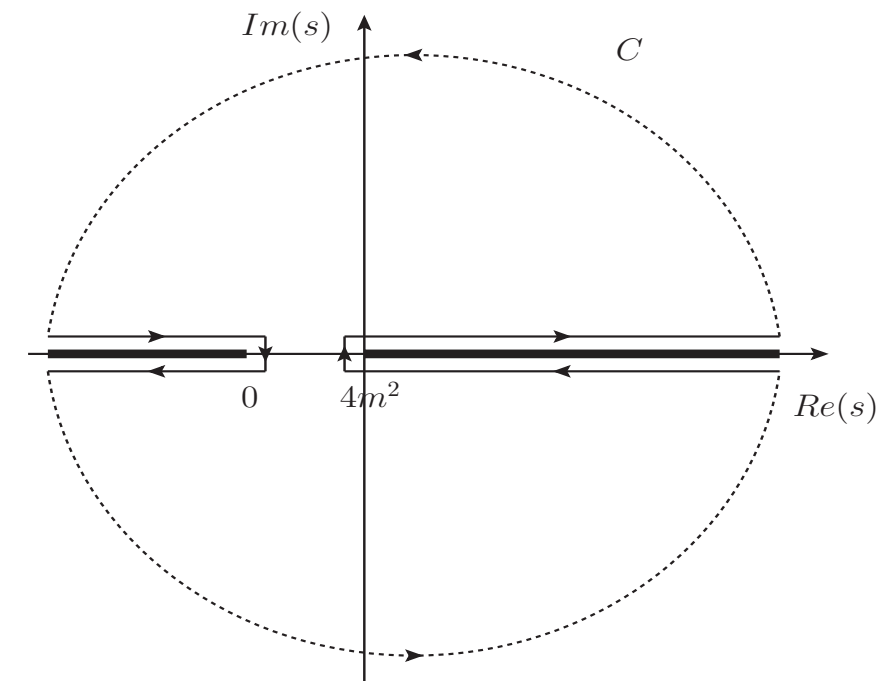
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Which we subtract once in accordance with **unitarity bound**

$$t_{ab}(s) = U_{ab}(s) + \frac{s - s_M}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s' - s_M} \frac{\text{Disc } t_{ab}(s')}{s' - s}$$



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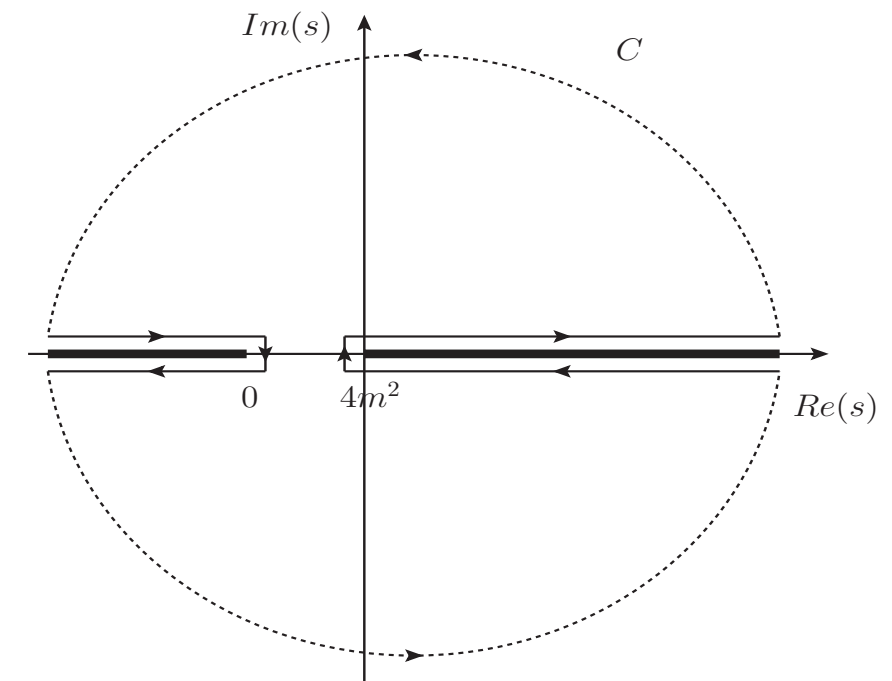
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included subtraction constant and left-hand cuts, asymptotically bounded **unknown function**



Once-subtracted p.w. dispersion relation

$$t_{ab}(s) = U_{ab}(s) + \frac{s}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{\text{Disc } t_{ab}(s')}{s' - s}$$

can be solved using N/D method with input from $U_{ab}(s)$ **above threshold**

$$t_{ab}(s) = \sum_c D_{ac}^{-1} N_{cb}(s)$$

$$N_{ab}(s) = U_{ab}(s) + \frac{s}{\pi} \sum_c \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{N_{ac}(s') \rho_c(s') (U_{cb}(s') - U_{cb}(s))}{s' - s}$$

$$D_{ab}(s) = \delta_{ab} - \frac{s}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{N_{ab}(s') \rho_b(s')}{s' - s} = \Omega_{ab}^{-1}(s)$$

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Omnès function

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need to solve integral equation

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Omnès function

Conformal mapping expansion

We approximate left-hand cuts as an expansion in a **conformal mapping variable** $\xi(s)$

Gasparyan, Luts, Nucl. Phys. A848, 126 (2010)

$$U_{ab}(s) = t_{ab}(0) + \frac{s}{\pi} \int_{-\infty}^{s_L} \frac{ds'}{s'} \frac{\text{Im } t_{ab}(s')}{s' - s} \simeq \sum_{n=0}^{\infty} C_{ab,n} \xi_{ab}^n(s)$$

to be determined from the fits

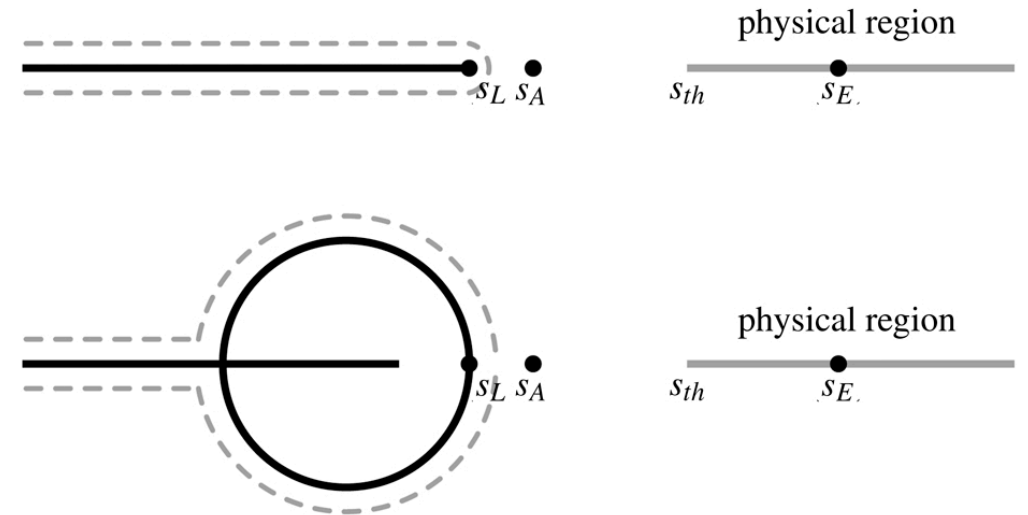
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The exact form of the conformal map

$$\{\pi\pi, K\bar{K}\} \quad \xi(s) = \frac{\sqrt{s - s_L} - \sqrt{s_E - s_L}}{\sqrt{s - s_L} + \sqrt{s_E - s_L}}$$

$$\xi(s_E) = 0$$

$$\sqrt{s_E} = \frac{1}{2} \left(\sqrt{s_{th}} + \sqrt{s_{max}} \right)$$

$$\pi K \quad \xi(s) = - \frac{(\sqrt{s} - \sqrt{s_E})(\sqrt{s}\sqrt{s_E} + s_L)}{(\sqrt{s} + \sqrt{s_E})(\sqrt{s}\sqrt{s_E} - s_L)}$$

source of the systematic uncertainties

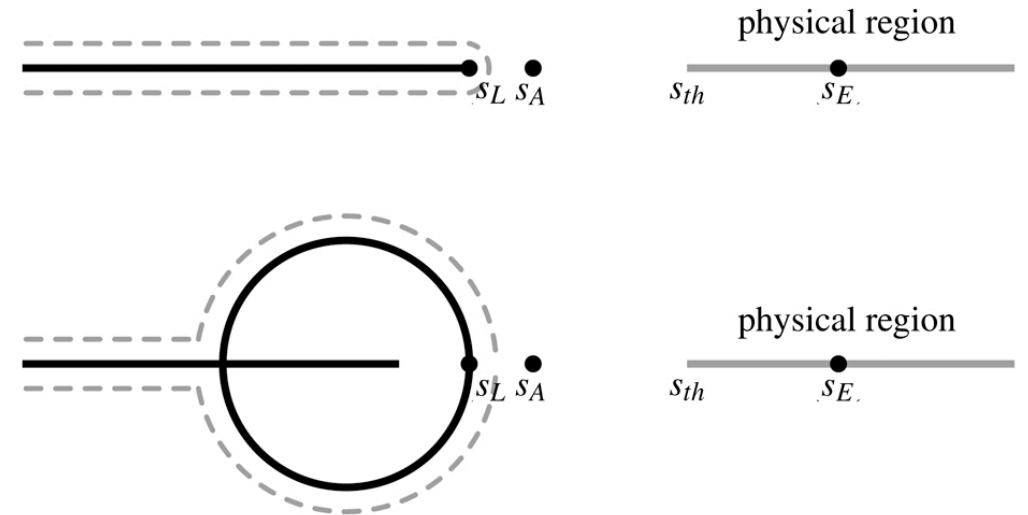
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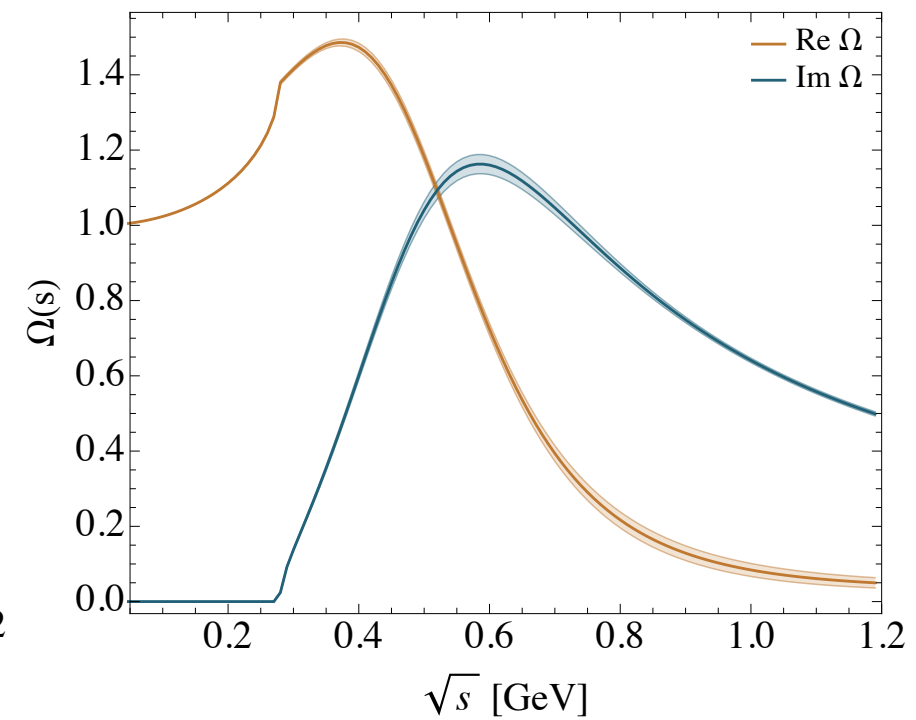
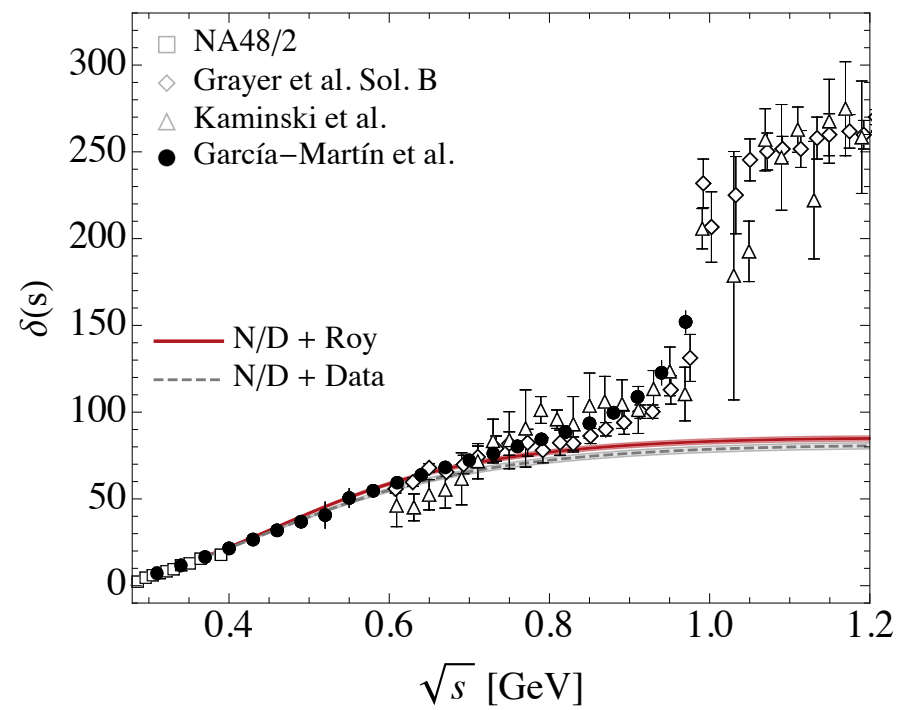
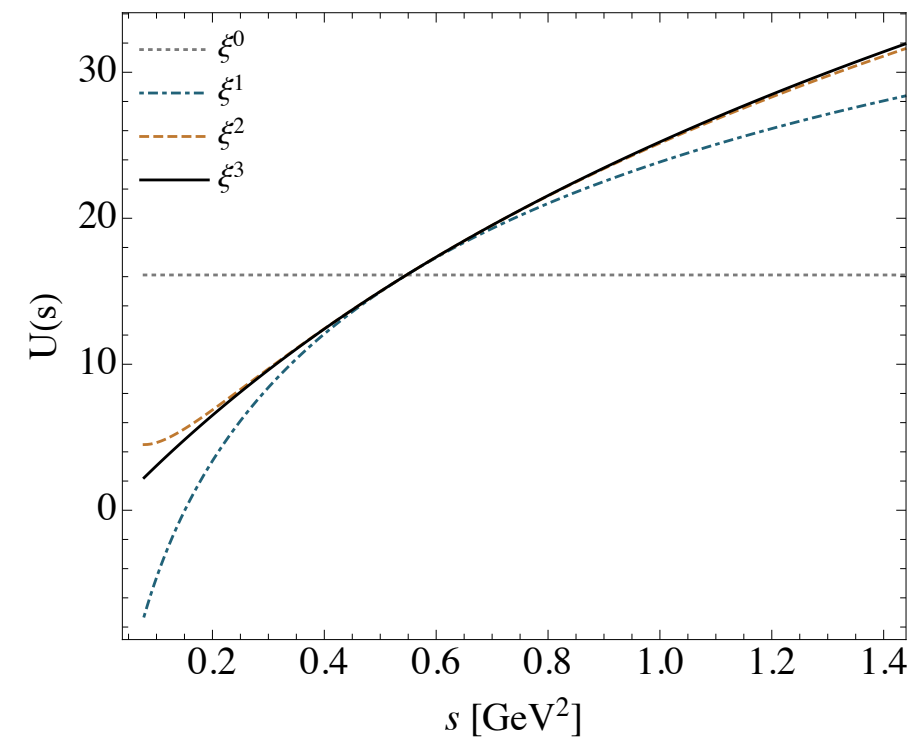
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source of the systematic uncertainties

N/D approach requires solving an integral equation \implies we use **parametric bootstrap** technique for fitting and error analysis

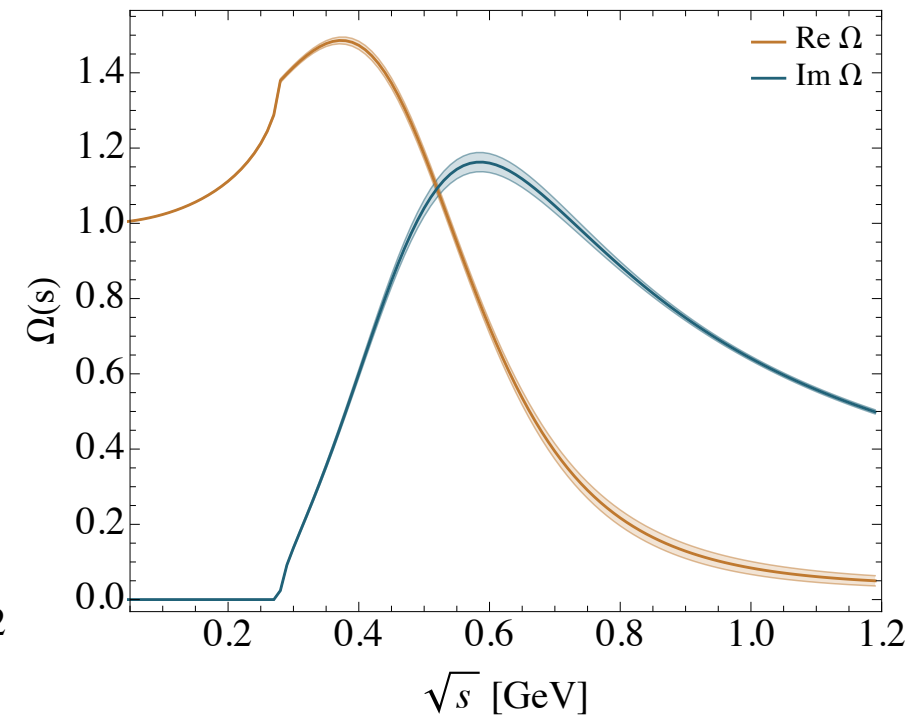
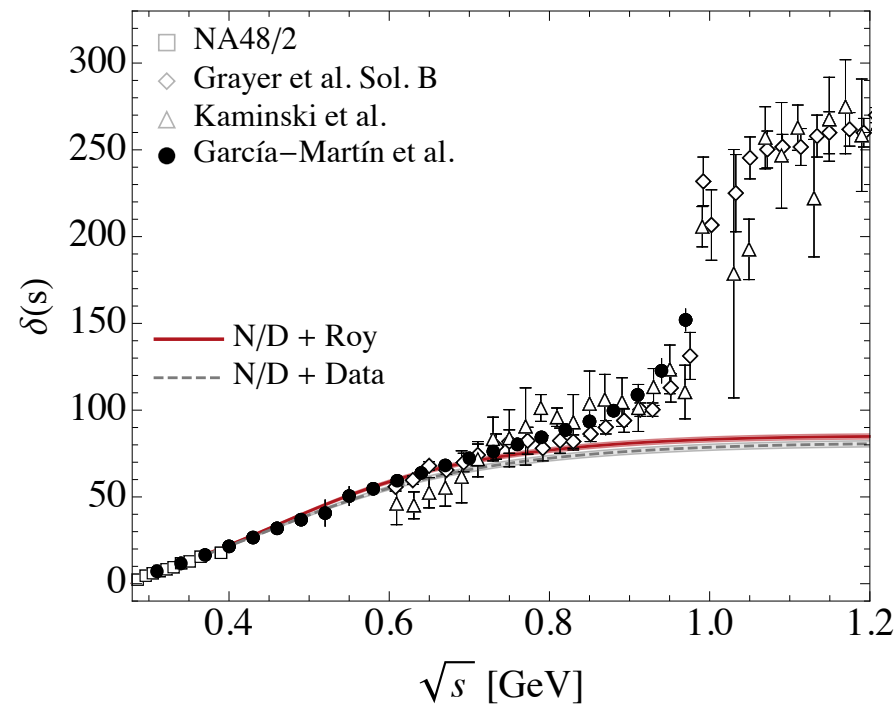
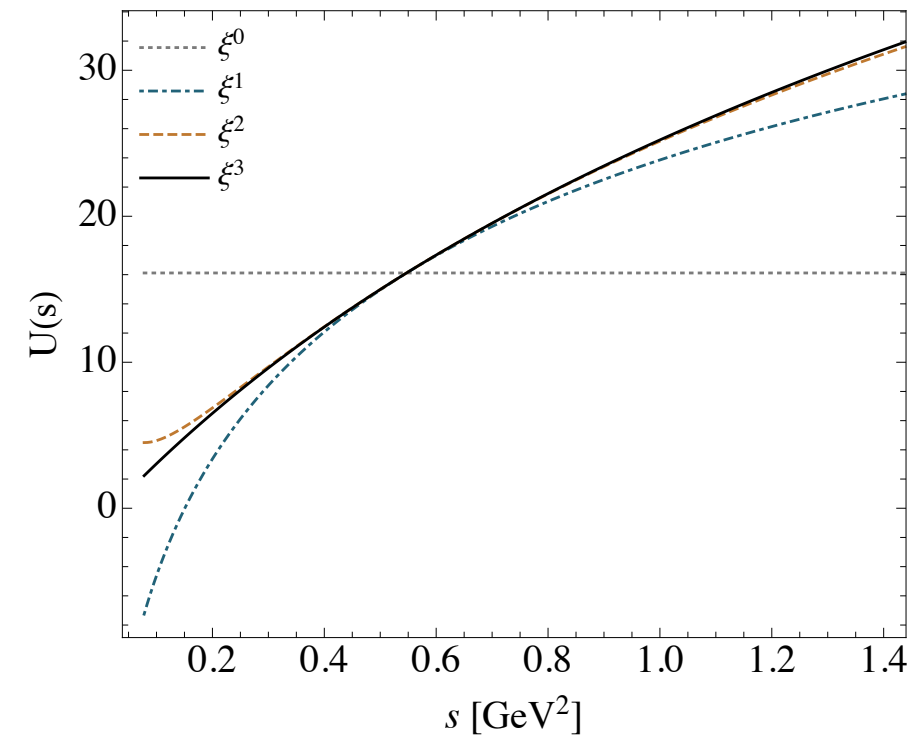
Single channel $\pi\pi$ scattering



Input: experimental data/Roy analysis + NNLO χ PT a, b + NLO χ PT Adler zero

4 parameters fit

Single channel $\pi\pi$ scattering



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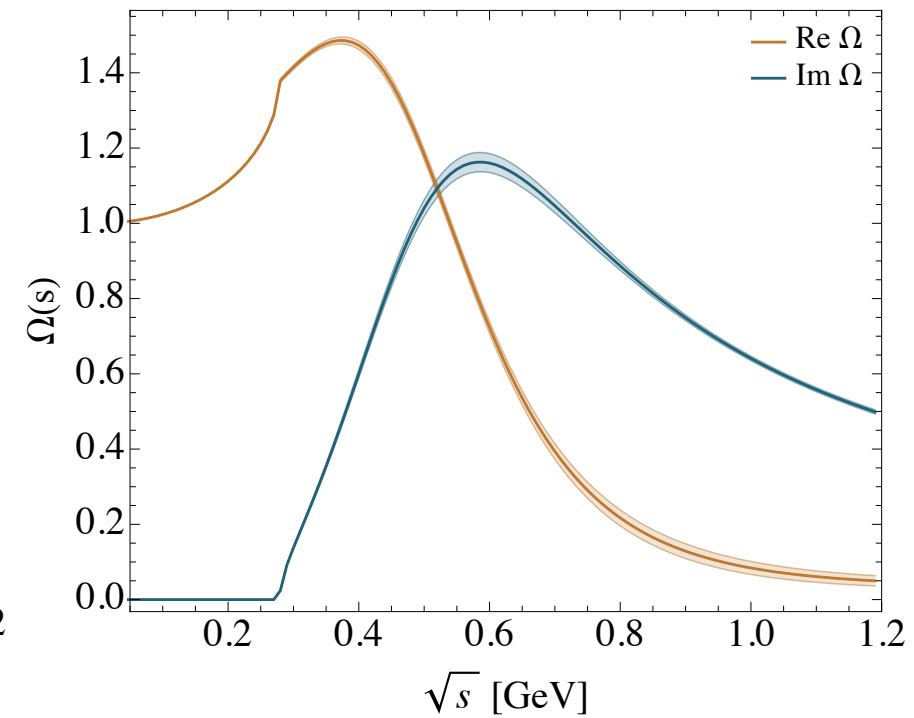
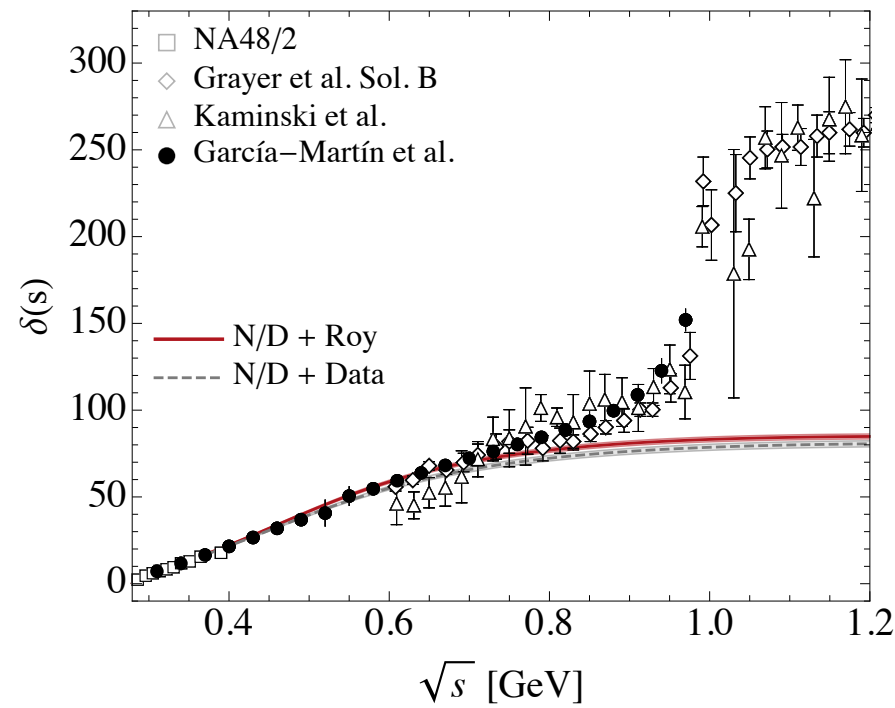
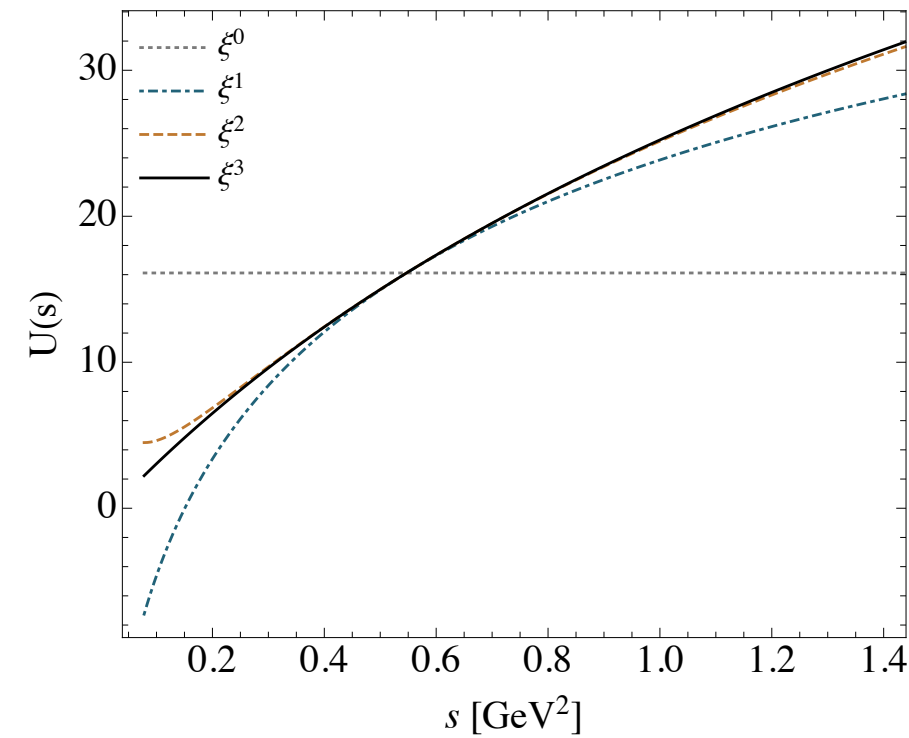
$$\sqrt{s_{f_0(500),ND+exp}} = 435(7)_{-8}^{+6} - i 250(5)_{-8}^{+6} \text{ MeV}$$

$$\sqrt{s_{f_0(500),ND+Roy}} = 458(7)_{-10}^{+4} - i 245(6)_{-10}^{+7} \text{ MeV}$$

$$\sqrt{s_{f_0(500),Roy}} = 449_{-16}^{+22} - i 275(15) \text{ MeV}$$

García-Martín *et al.* Phys.Rev.D 83 (2011) 074004
 Caprini *et al.* Phys. Rev. Lett. 96, 132001 (2006)

Single channel $\pi\pi$ scattering



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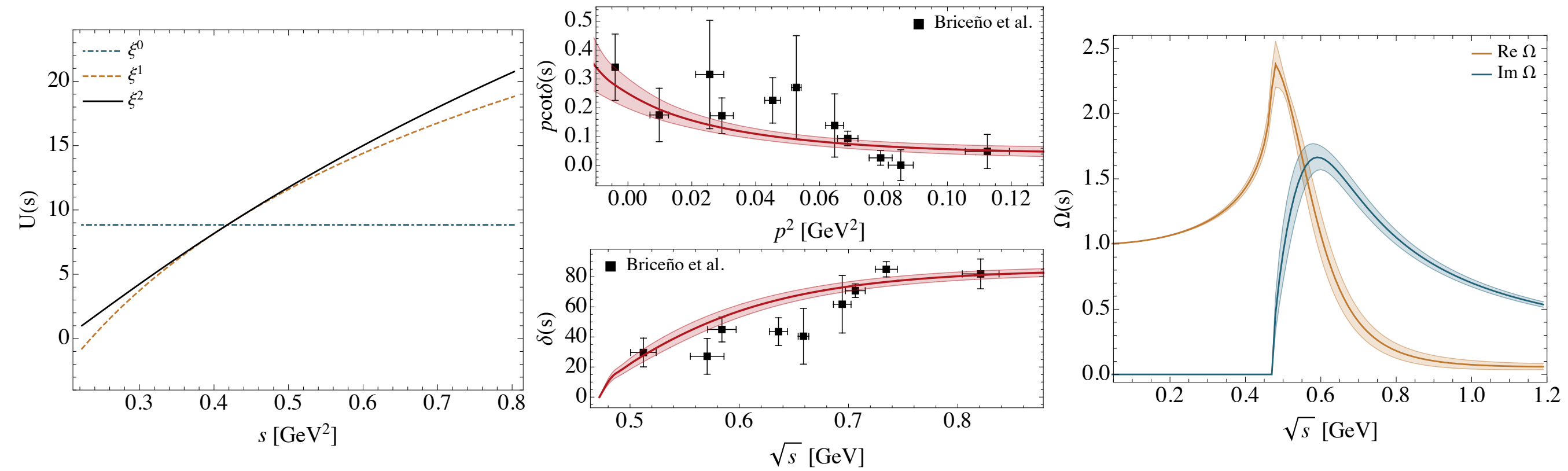
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$$\Omega(s) = D^{-1}(s) = \exp \left(\frac{s}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{\delta(s')}{s' - s} \right)$$

Similar results for single channel $\pi\pi$ phase-shift and Omnès can be obtained by using mlAM with the χ PT input

Gomez Nicola *et al.* Phys. Rev. D 77, 056006 (2008)

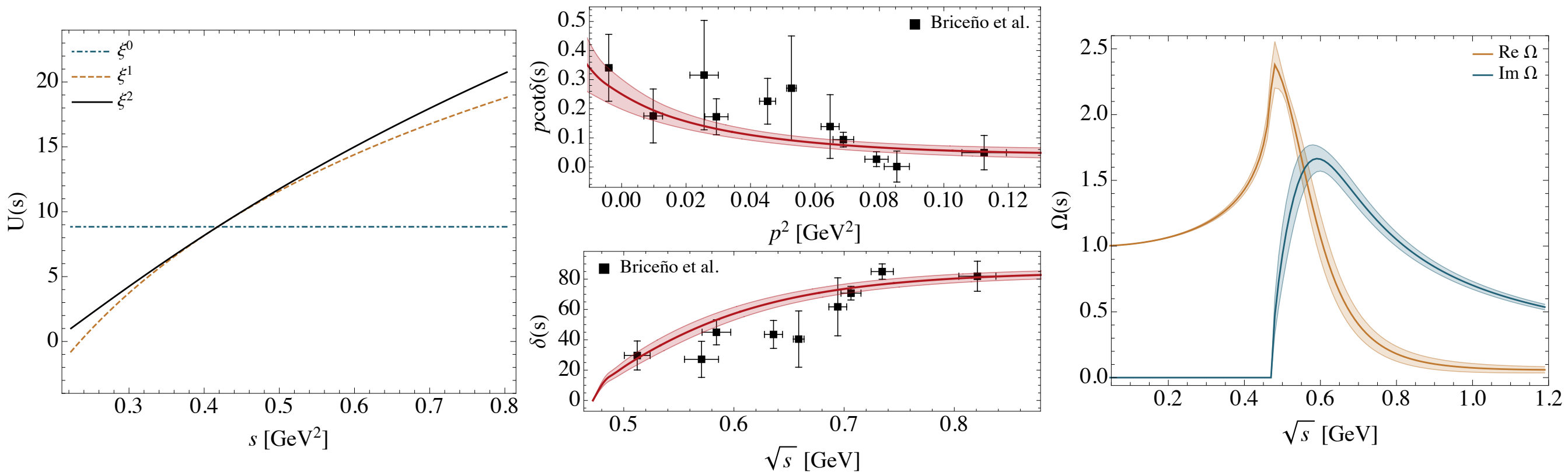
Single channel $\pi\pi$ scattering for $m_\pi = 236$ MeV



Input: Lattice data + NLO χ PT Adler zero

3 parameters fit

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Input: Lattice data + NLO χ PT Adler zero

3 parameters fit

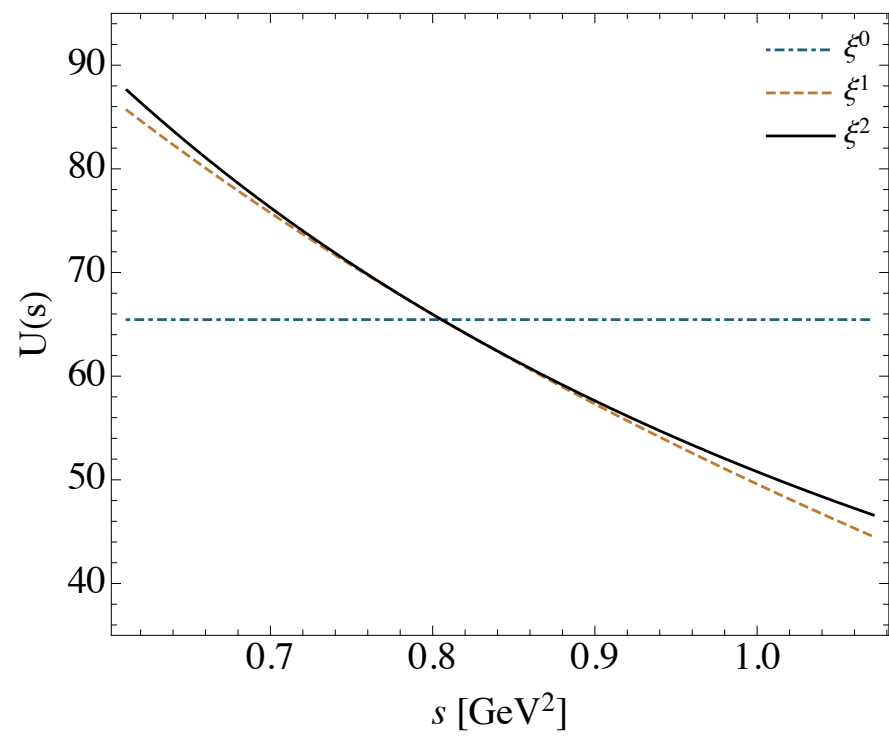
N/D analysis of the lattice data: $\sqrt{s_{f_0(500),ND+Latt}} = 498(21)_{-19}^{+12} - i 138(13)_{-10}^{+5}$ MeV

Briceño et al. Phys.Rev.Lett. 118 (2017) 2, 022002

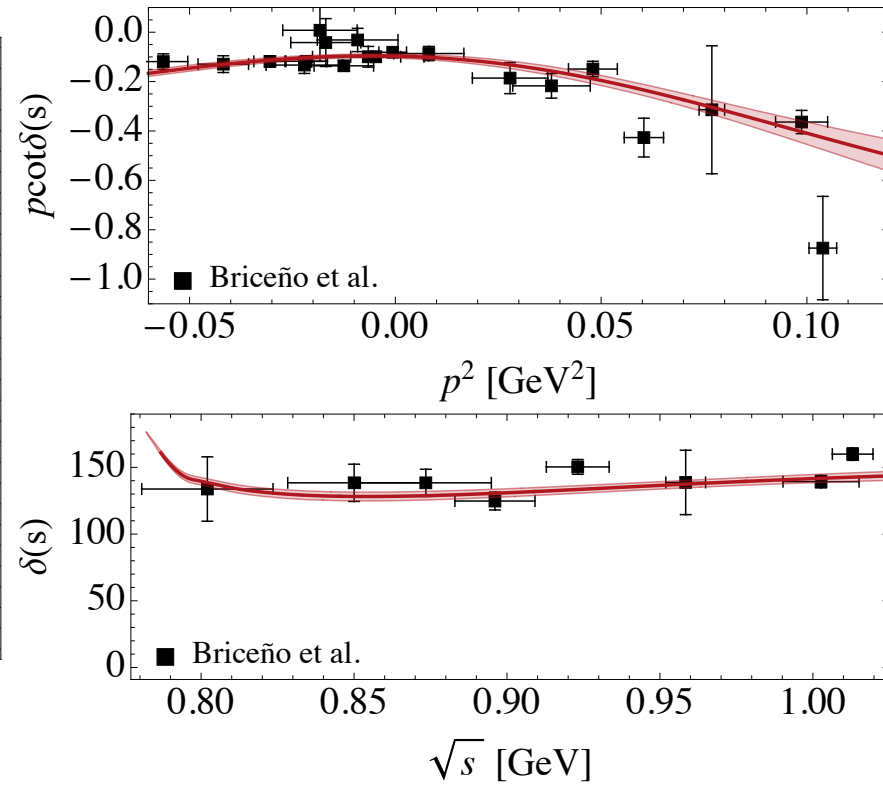
Predictions: $\sqrt{s_{f_0(500),mIAM_{NNLO}}} = 510 - i 175$ MeV Peláez, Rios Phys.Rev.D 82 (2010) 114002

$\sqrt{s_{f_0(500),BSE_{NLO}}} = 490(15) - i 180(10)$ MeV Albaladejo, Oller Phys. Rev. D86, 034003 (2012)

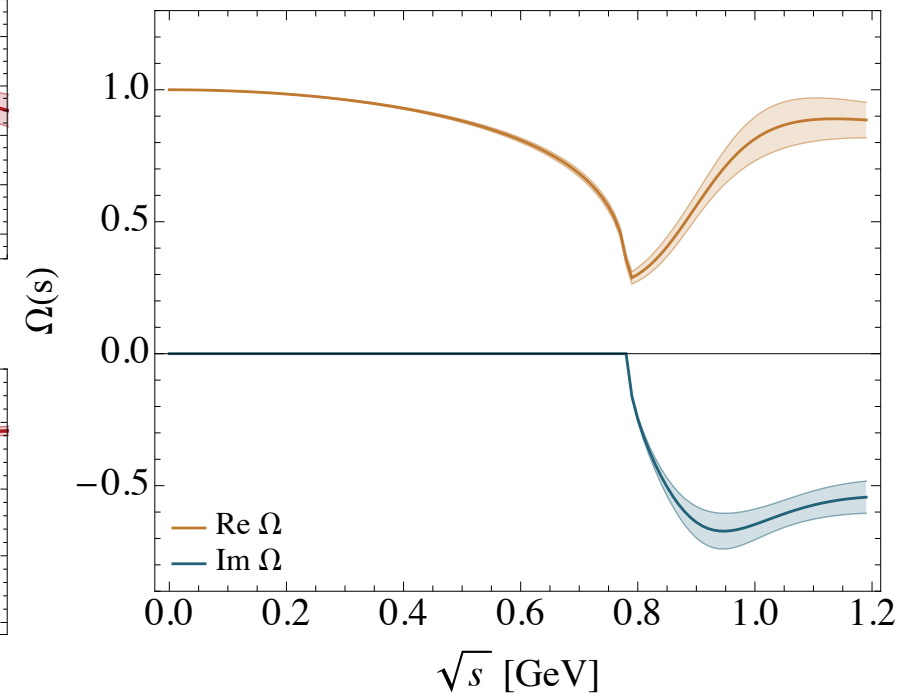
Single channel $\pi\pi$ scattering for $m_\pi = 391$ MeV



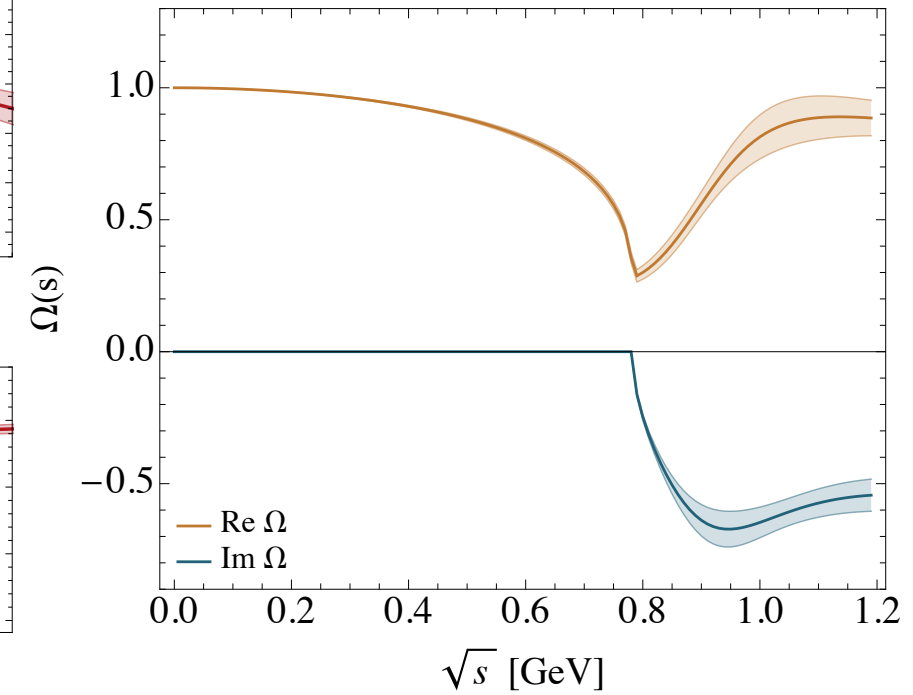
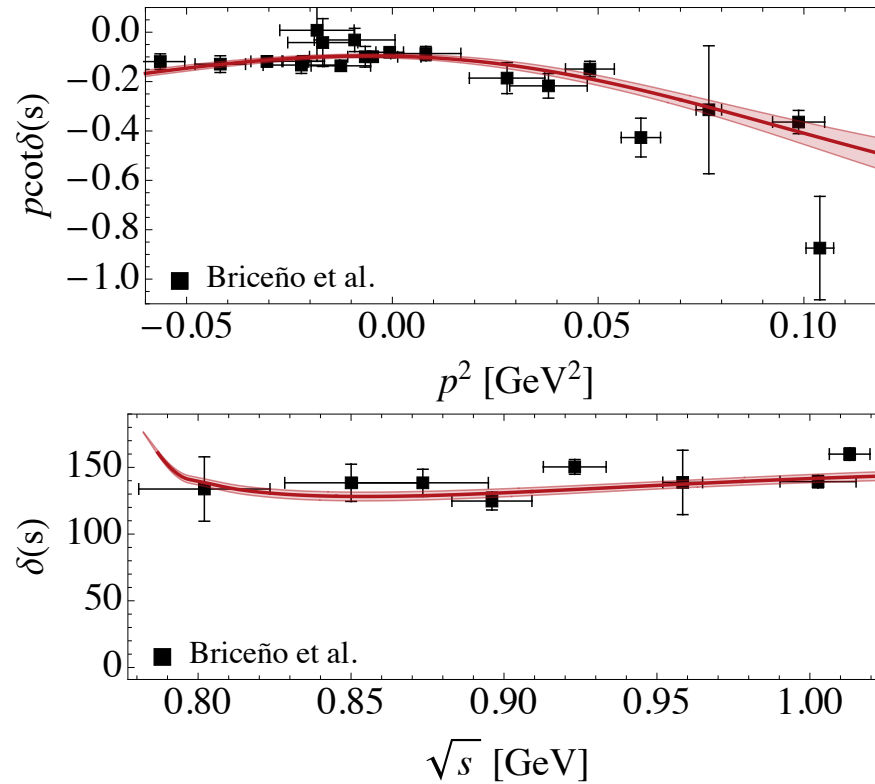
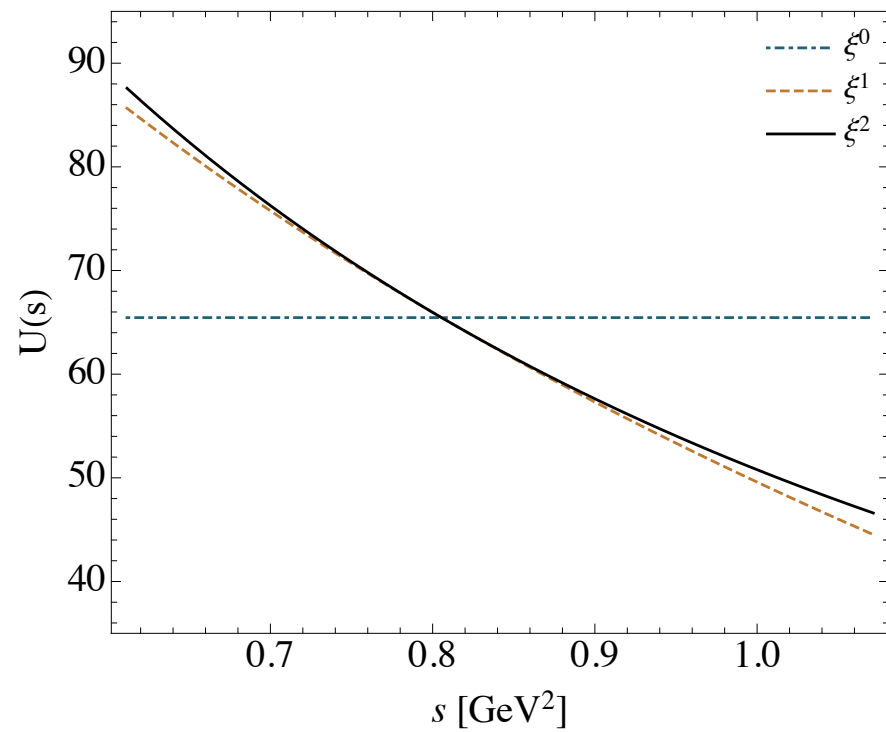
Input: Lattice data



3 parameters fit



Single channel $\pi\pi$ scattering for $m_\pi = 391$ MeV



Input: Lattice data

3 parameters fit

N/D analysis of the
lattice data:

$$\sqrt{s_{B,ND+Latt}} = 758(5)(0) \text{ MeV}$$

K-matrix analysis of
the lattice data:

$$\sqrt{s_{B,Kmat+Latt}} = 758(4) \text{ MeV}$$

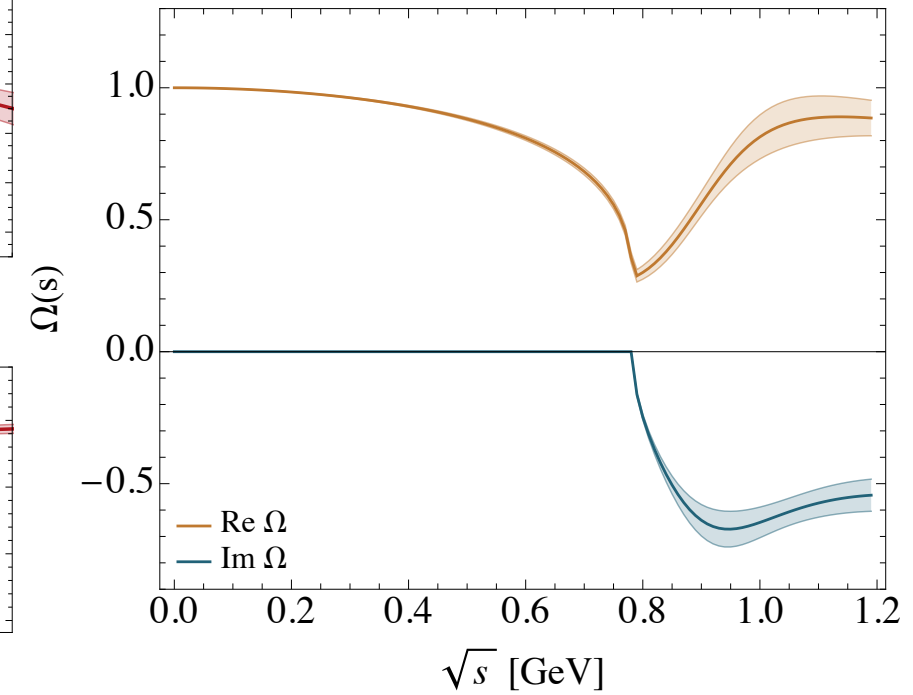
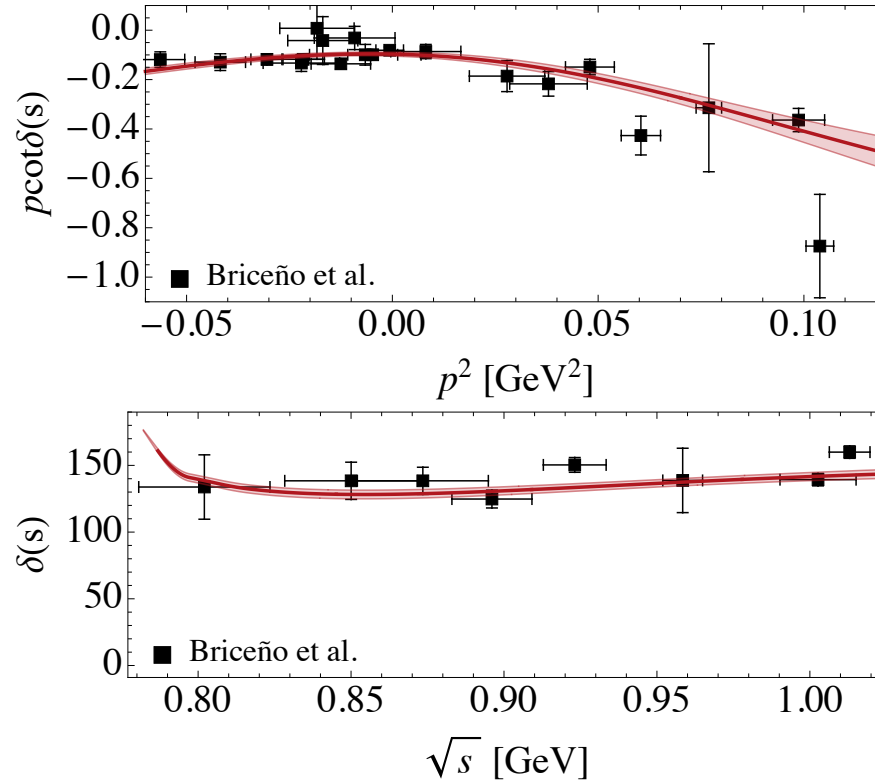
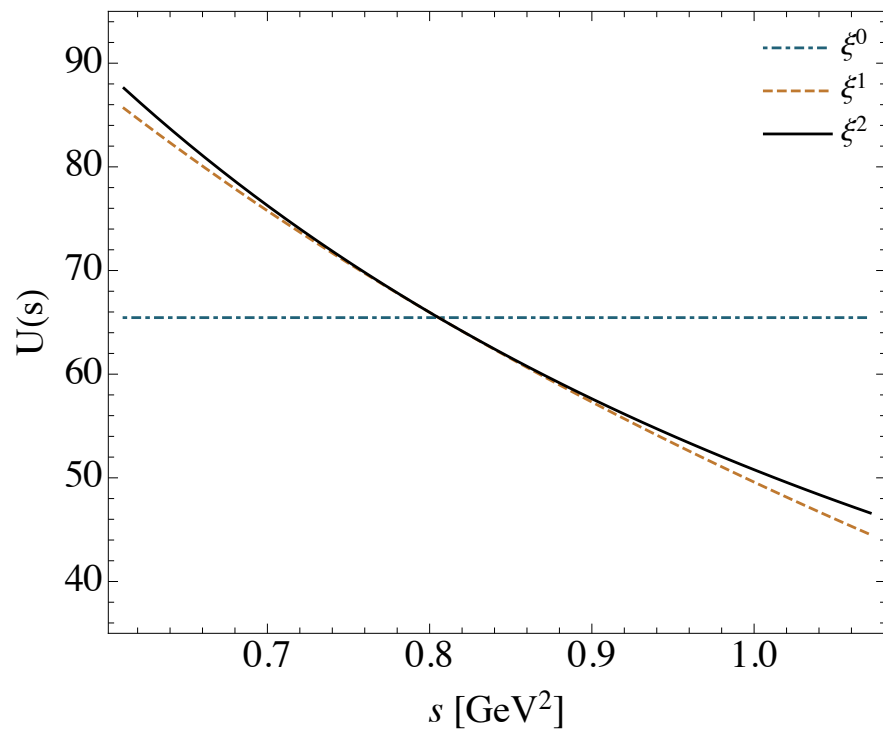
Prediction:

$$\sqrt{s_{B,mIAM_{NNLO}}} = 765 \text{ MeV}$$

Briceño *et al.* Phys.Rev.Lett. 118 (2017) 2, 022002

Peláez, Rios Phys.Rev.D 82 (2010) 114002

Single channel $\pi\pi$ scattering for $m_\pi = 391$ MeV



Input: Lattice data

3 parameters fit

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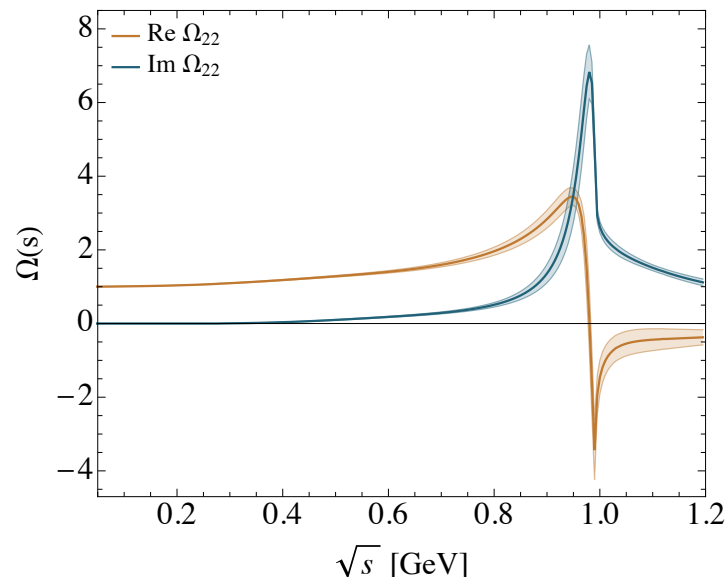
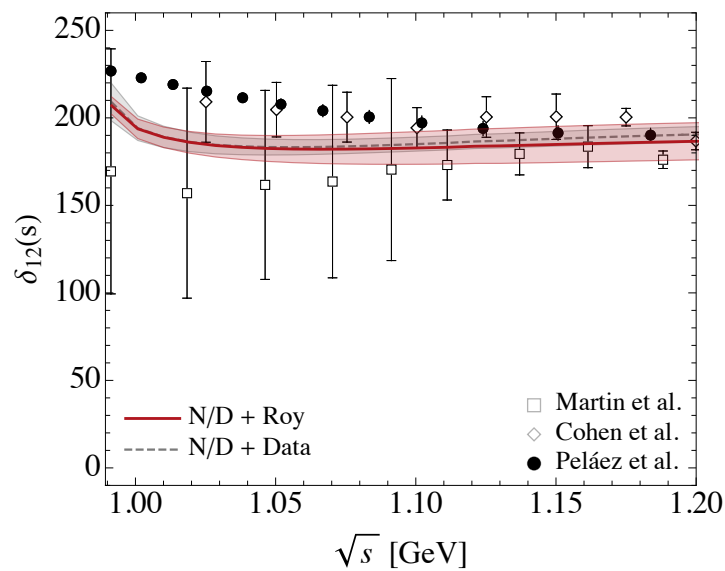
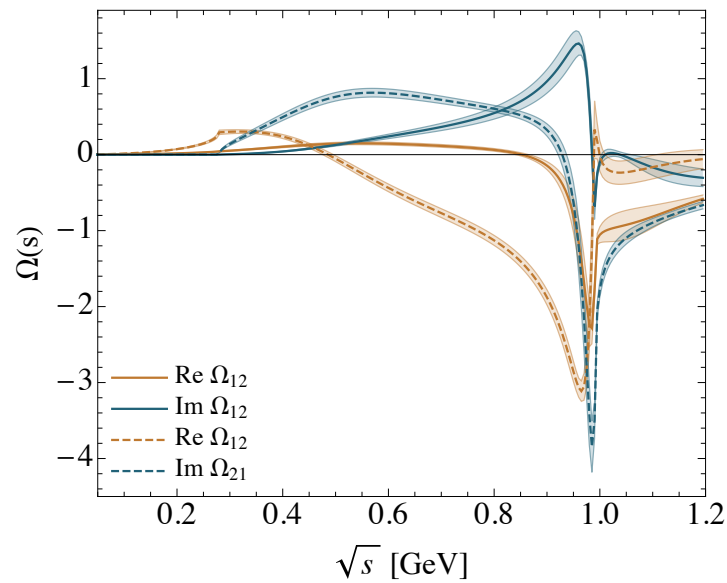
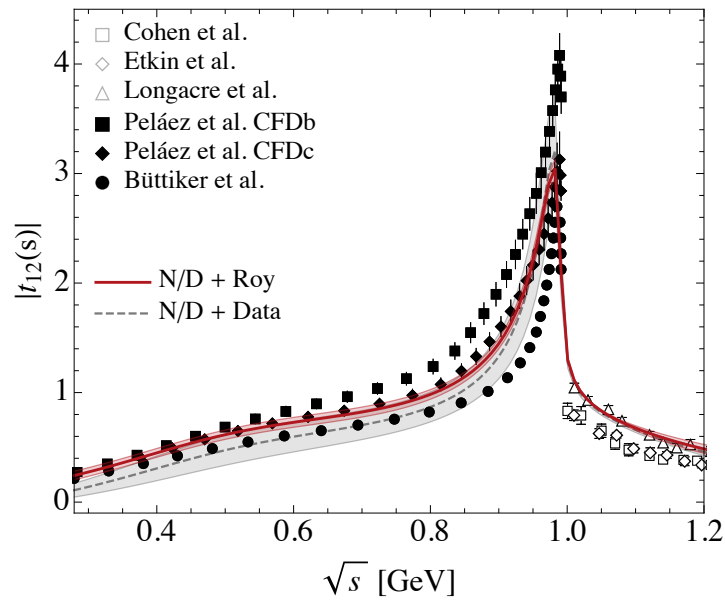
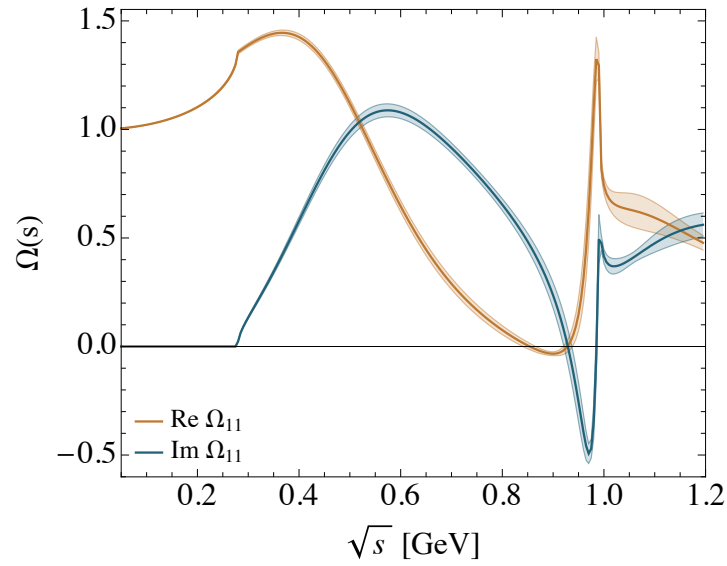
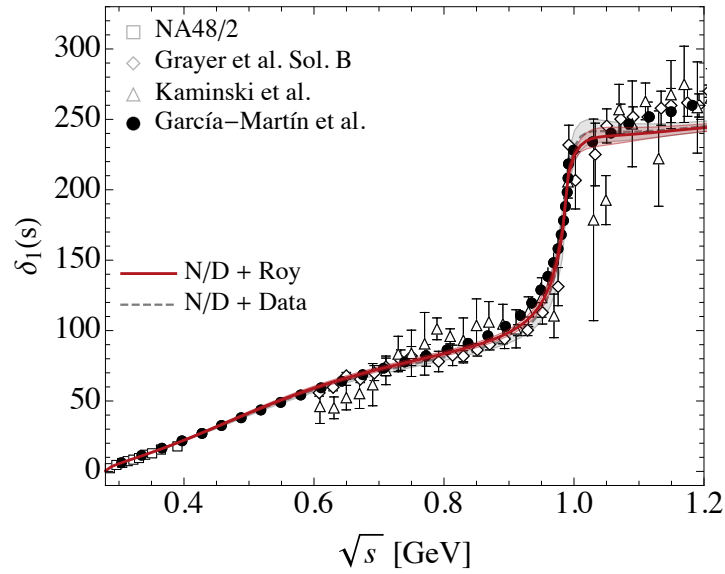
Briceño et al. Phys.Rev.Lett. 118 (2017) 2, 022002

Peláez, Rios Phys.Rev.D 82 (2010) 114002

$$\det(D_{ab}(s_B)) = 0 \quad s_B < s_{th}$$

$$t_{ab}(s) = U_{ab}(s) + \frac{s}{s_B} \frac{g_{Ba}g_{Bb}}{s_B - s} + \frac{s}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'} \frac{\text{Disc } t_{ab}(s')}{s' - s}$$

Coupled channel $\pi\pi$ scattering



$$t(s) = \begin{pmatrix} \frac{\eta(s) e^{2i\delta_1(s)} - 1}{2i\rho_1(s)} & |t_{12}(s)| e^{\delta_{12}(s)} \\ |t_{12}(s)| e^{\delta_{12}(s)} & \frac{\eta(s) e^{2i\delta_2(s)} - 1}{2i\rho_2(s)} \end{pmatrix}$$

$$\delta_{12}(s) = \delta_1(s) + \delta_2(s) \theta(s > 4m_K^2)$$

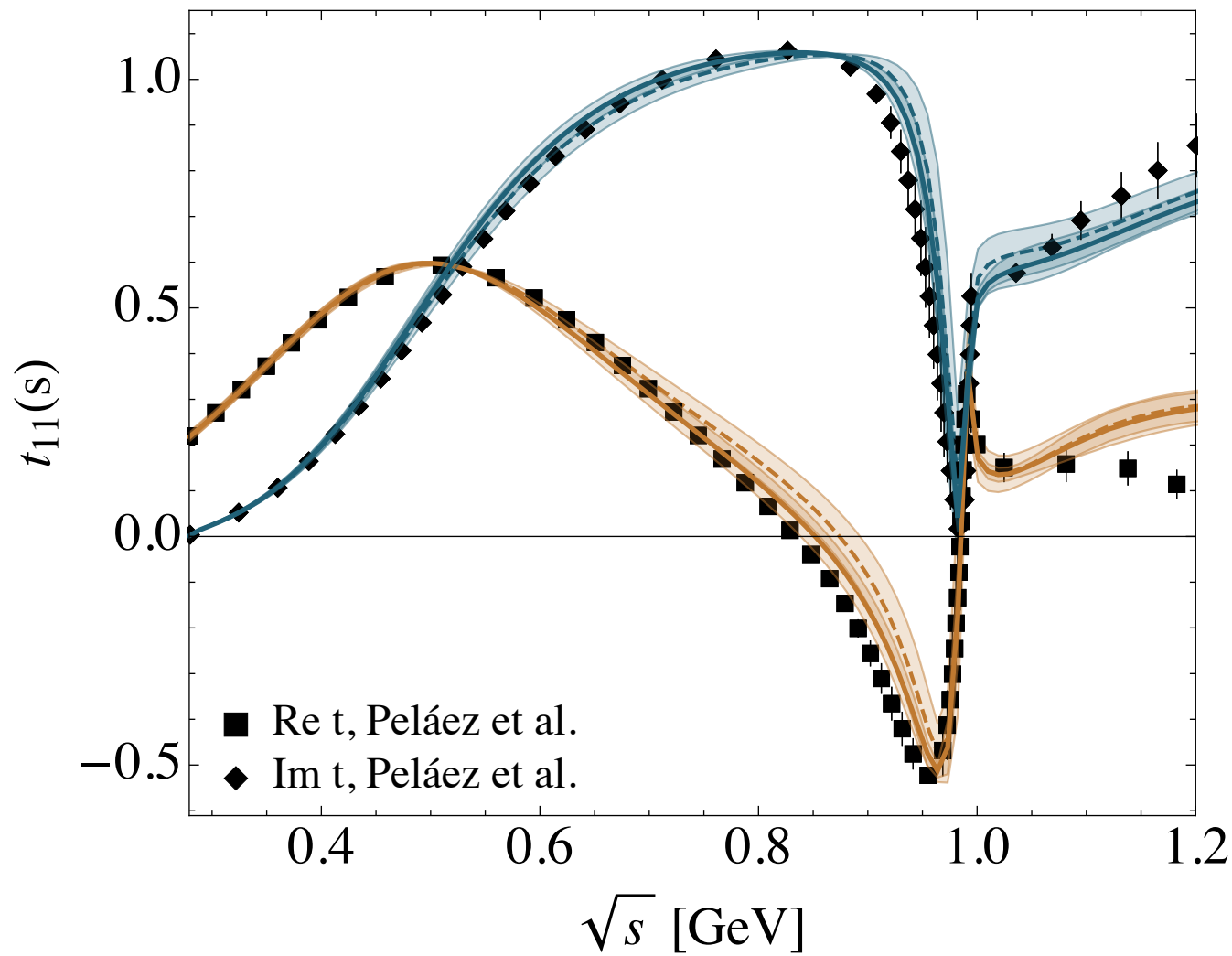
$$\eta(s) = \sqrt{1 - 4\rho_1(s)\rho_2(s)|t_{12}(s)|^2}$$

4+2+3 parameters fit

García-Martín et al. Phys.Rev.D 83 (2011) 074004

Peláez, Rodas arXiv: 2010.11222 [hep-ph]

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$$\delta_{12}(s) = \delta_1(s) + \delta_2(s) \theta(s > 4m_K^2)$$

4+2+3 parameters fit

$$\sqrt{s_{f_0(500),ND+exp}} = 454(12)_{-7}^{+6} - i262(12)_{-12}^{+8} \text{ MeV}$$

$$\sqrt{s_{f_0(500),ND+Roy}} = 458(10)_{-15}^{+7} - i256(9)_{-8}^{+5} \text{ MeV}$$

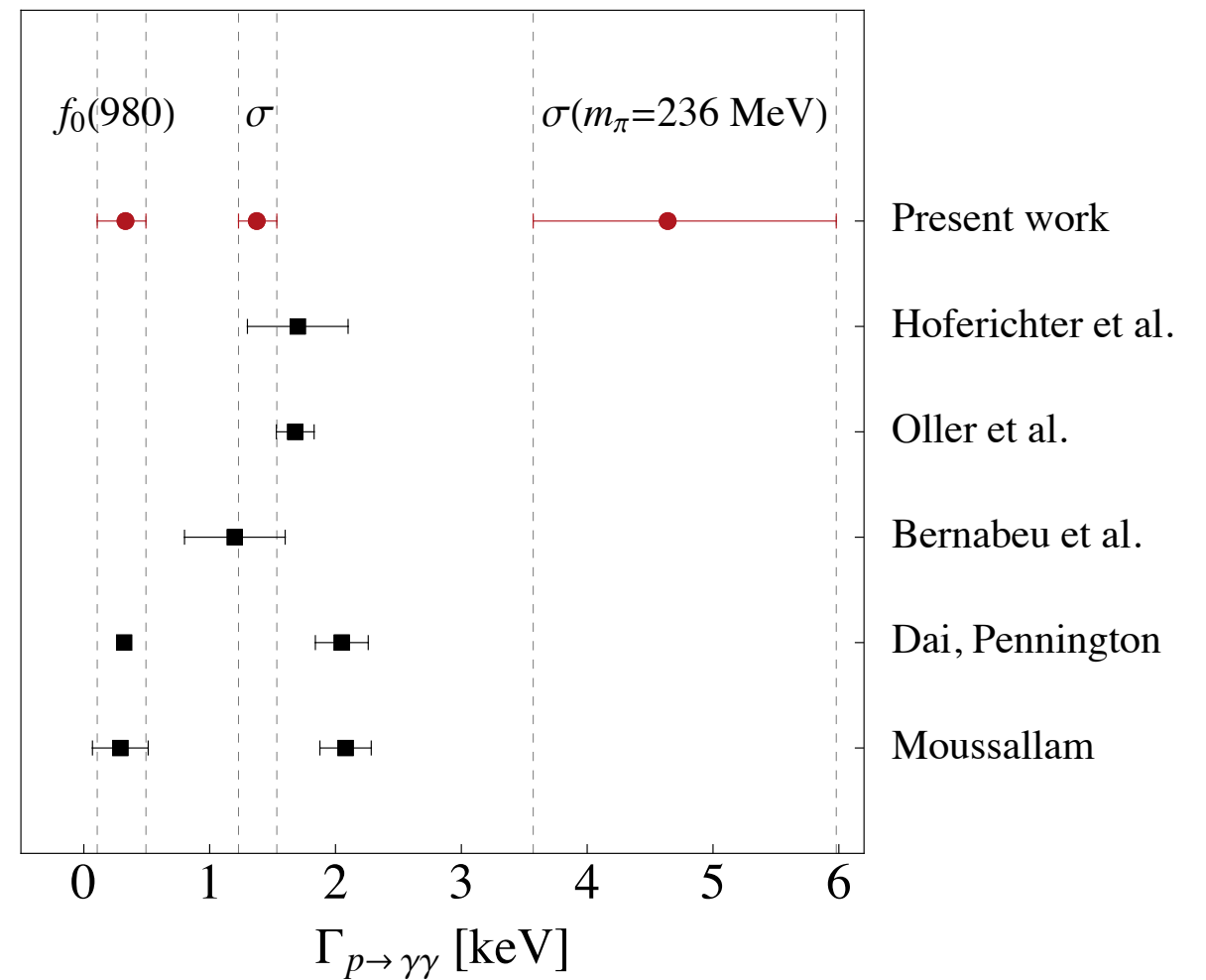
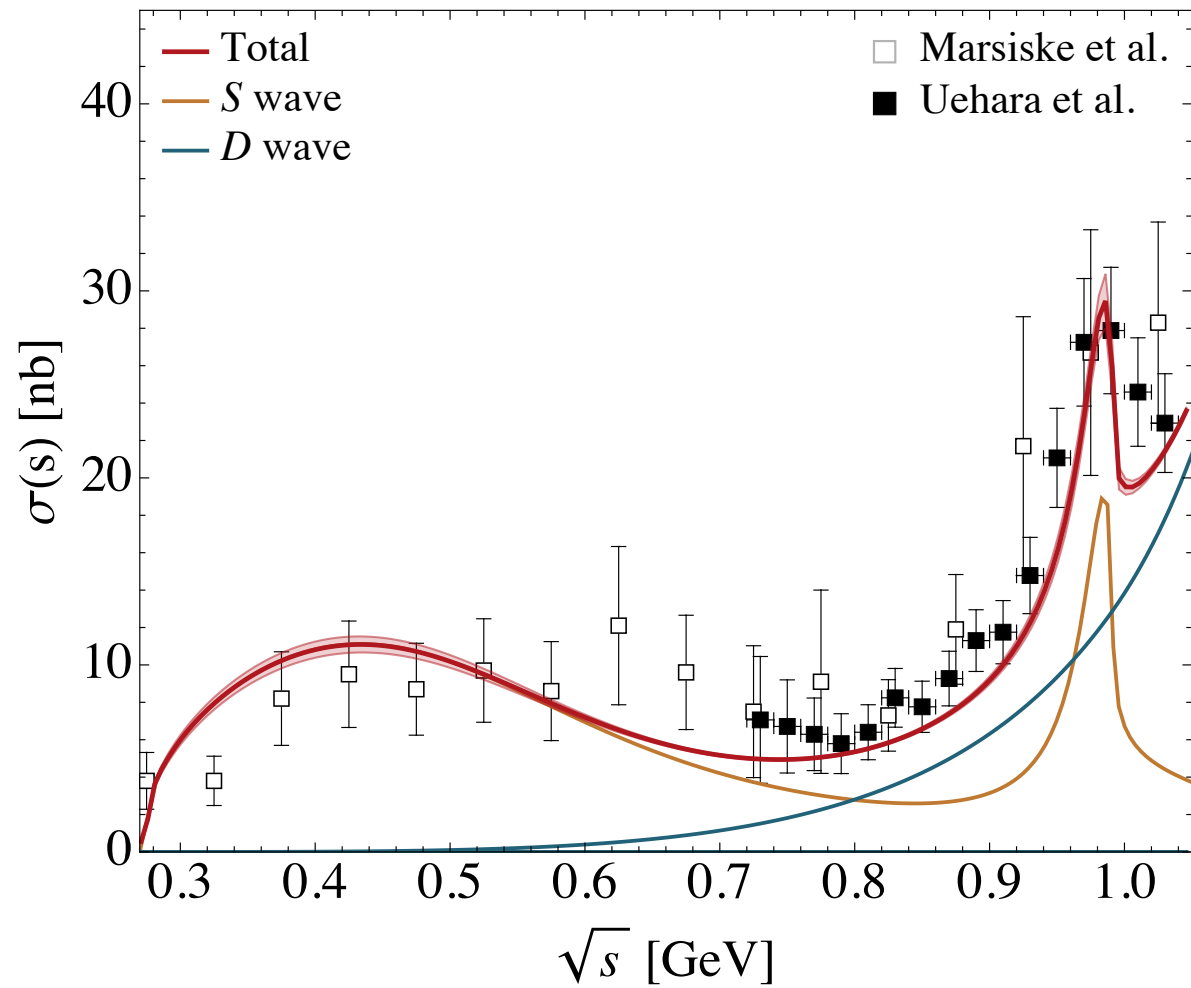
$$\sqrt{s_{f_0(500),Roy}} = 449_{-16}^{+22} - i245(15) \text{ MeV}$$

$$\sqrt{s_{f_0(980),ND+exp}} = 990(7)_{-4}^{+2} - 17(7)_{-1}^{+4} \text{ MeV}$$

$$\sqrt{s_{f_0(980),ND+Roy}} = 993(2)_{-1}^{+2} - i21(3)_{-4}^{+2} \text{ MeV}$$

$$\sqrt{s_{f_0(980),Roy}} = 996_{-14}^{+7} - i25_{-6}^{+11} \text{ MeV}$$

Application to $\gamma\gamma \rightarrow \pi\pi$ process

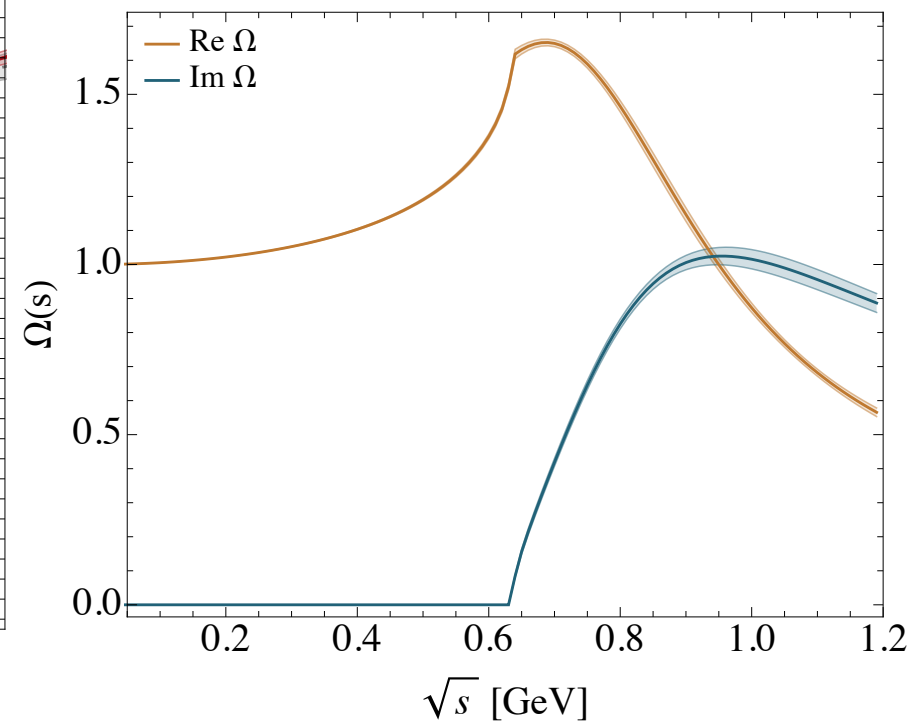
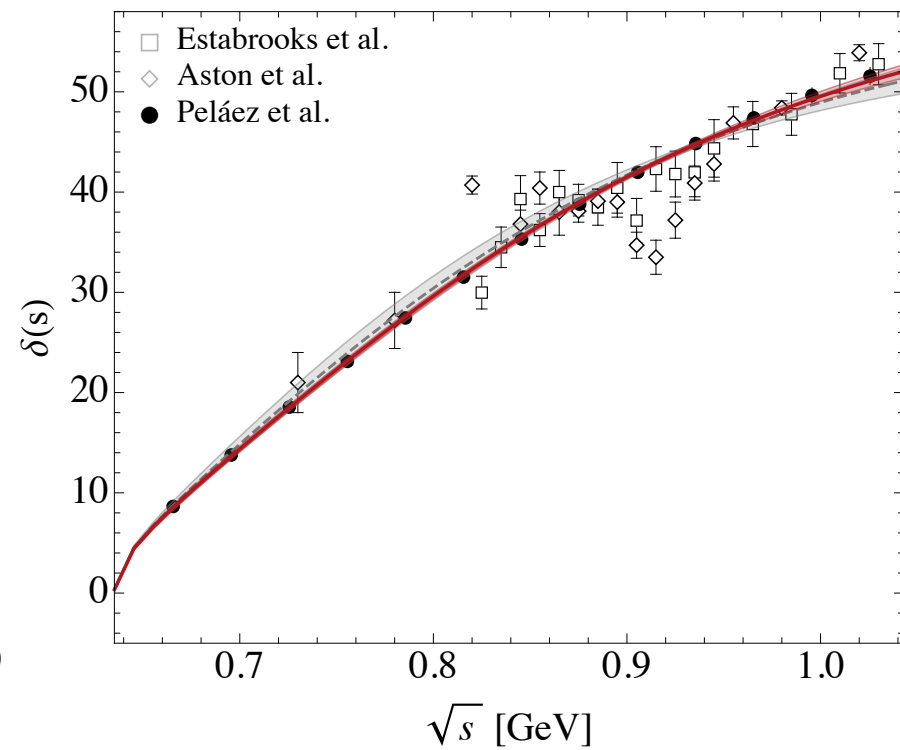
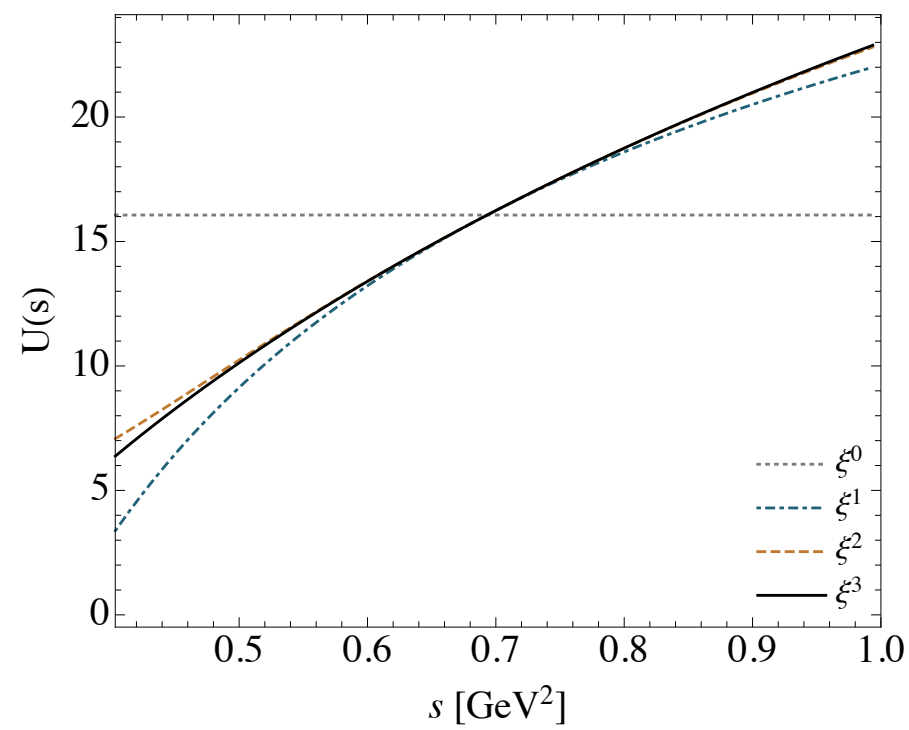


$$\Gamma_{\sigma \rightarrow \gamma\gamma} = 1.37(13)^{+0.09}_{-0.06} [1.38(9)^{+0.01}_{-0.01}] \text{ keV}$$

$$\Gamma_{f_0(980) \rightarrow \gamma\gamma} = 0.33(16)^{+0.04}_{-0.16} \text{ keV}$$

$$\Gamma_{\sigma \rightarrow \gamma\gamma}^{m_\pi=236} = 4.64(1.01)^{+0.88}_{-0.35} \text{ keV}$$

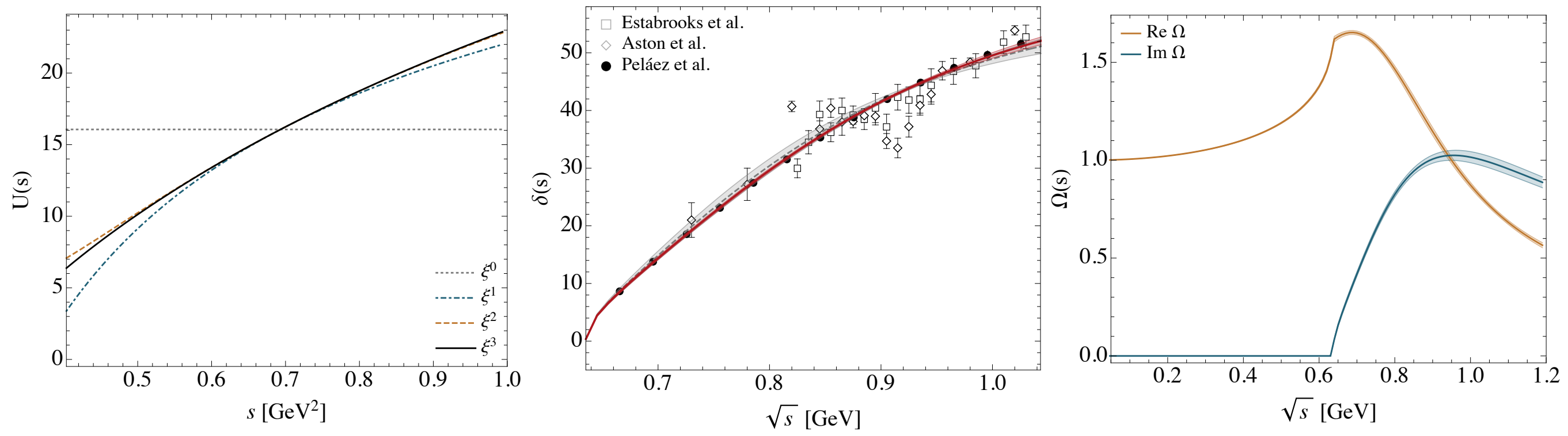
Single channel πK scattering



Input: experimental data/Roy analysis + NNLO χ PT a + NLO χ PT Adler zero

4 parameters fit

Single channel πK scattering



Input: experimental data/Roy analysis + NNLO χ PT a + NLO χ PT Adler zero

4 parameters fit

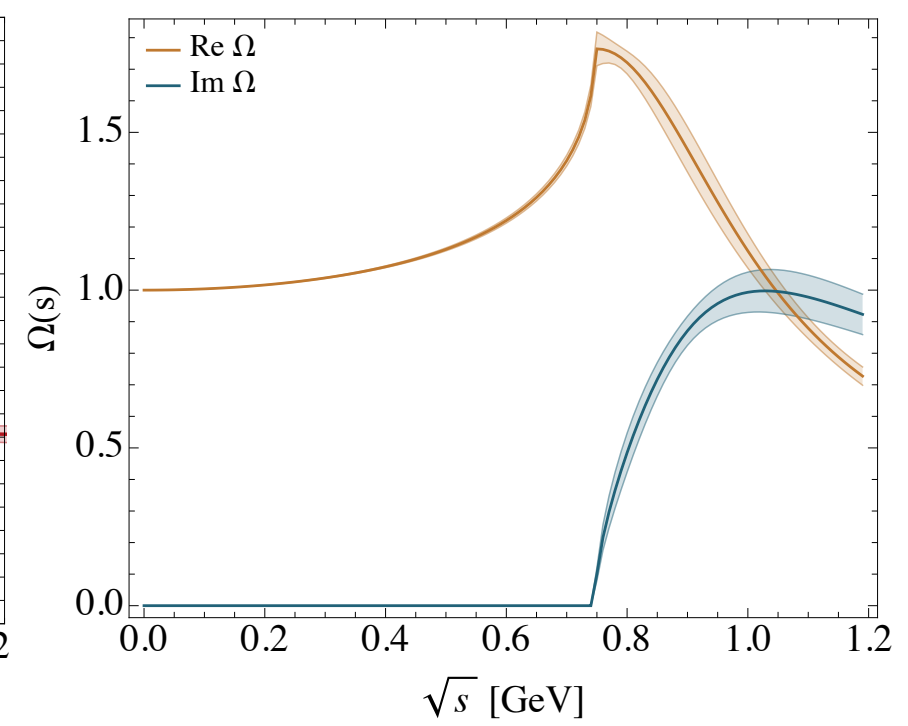
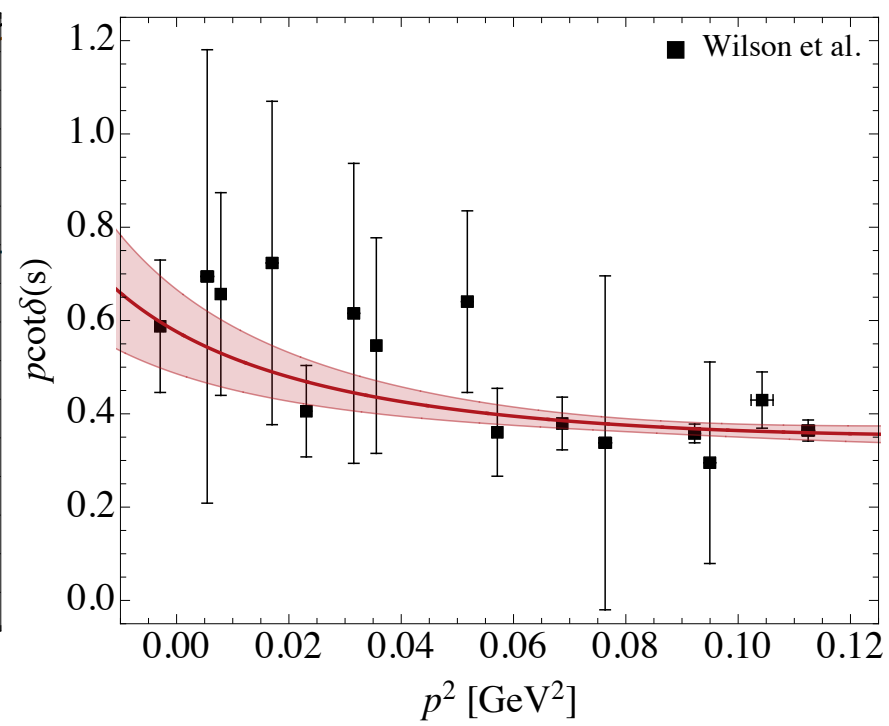
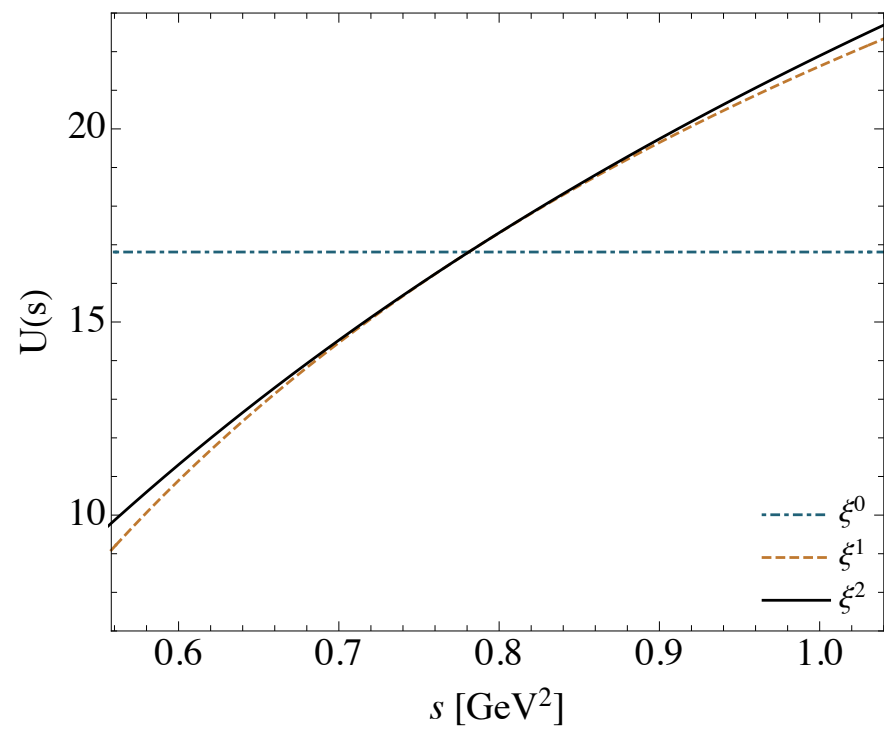
$$\sqrt{s_{K_0^*(700), ND+exp}} = 689(24)_{-2}^{+3} - i 263(33)_{-8}^{+5} \text{ MeV}$$

$$\sqrt{s_{K_0^*(700), ND+Roy}} = 702(12)_{-5}^{+4} - i 285(16)_{-13}^{+8} \text{ MeV}$$

$$\sqrt{s_{K_0^*(700), Roy}} = 653_{-12}^{+18} - i 280(16) \text{ MeV}$$

Peláez, Rodas Phys. Rev. Lett. 124 (2020) 17, 172001

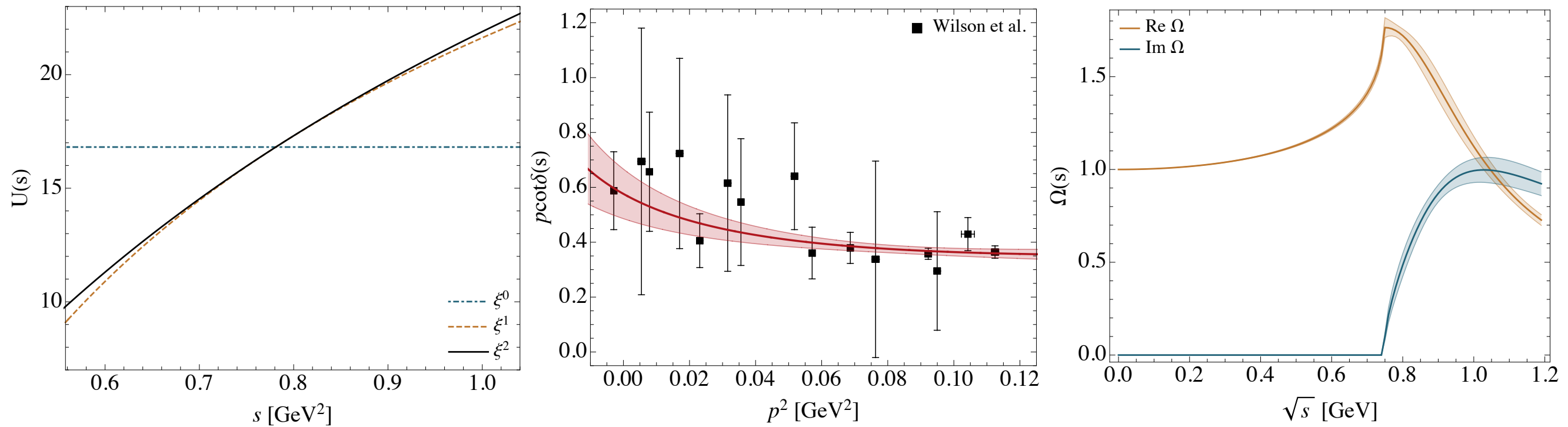
Single channel πK scattering for $m_\pi = 239 \text{ MeV}$



Input: lattice data + NLO χ PT Adler zero

3 parameters fit

Single channel πK scattering for $m_\pi = 239$ MeV



Input: lattice data + NLO χ PT Adler zero

3 parameters fit

$$\sqrt{s_{f_0(500),ND}} = 747(39)_{-0}^{+2} - i 265(16)_{-6}^{+7} \text{ MeV}$$

Wilson et al. Phys. Rev. Lett. 123, 042002 (2019)

This result is compatible qualitatively with the prediction from $U\chi$ PT

Nebreda, Peláez Phys. Rev.D81, 054035 (2010).

Summary and outlook

- We constructed a data driven dispersive approach based on the N/D ansatz, which can be applied both to the experimental and lattice data.
- We are able to describe $\sigma/f_0(500)$, $f_0(980)$, $\kappa/K^*(700)$ resonances. Our analysis provides consistent result whether the data or Roy/analysis input is used which allows to extend the range of applicability to the processes, where no Roy analysis is available.
- We offer a consistent tool, which fully respects unitarity and analyticity and can be an alternative to the K-matrix approach for the analysis of the lattice data
- The current result for the $f_0(980)$ resonance has been used for the calculation of its contribution to muon ($g - 2$)
- This method can also be extended for the coupled-channel analysis of the lattice data
- There are many exciting processes to which the method can be applied, e.g $J/\psi J/\psi$ scattering, for which the data from LHC is available

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Thank you!

Supplementary

