



Ordinary and exotic mesons from Dyson-Schwinger equations

Mini-Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) [2008.10240]

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [2004.00415]

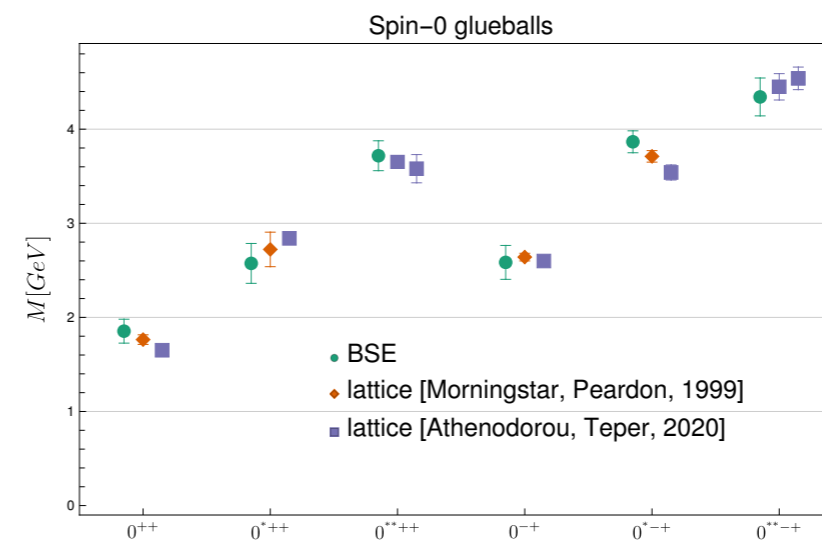
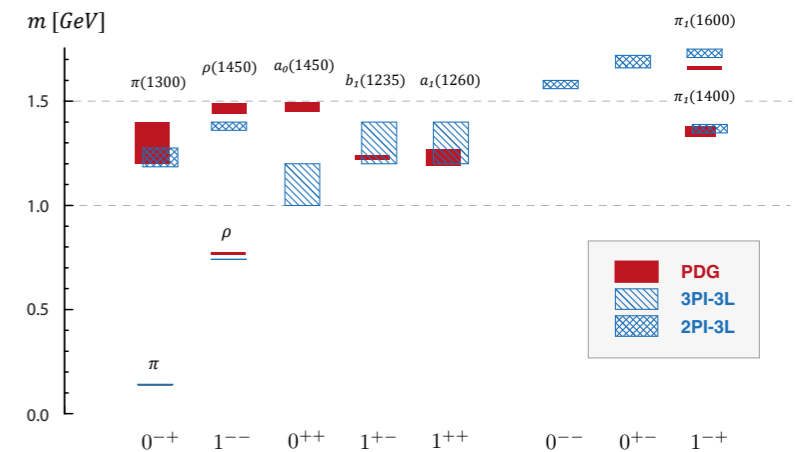
1. Light states: glueballs, two-quark and four-quark states

Williams, CF, Heupel, PRD 93 (2016) 034026 [arXiv:1512.00455]

Eichmann, CF, Heupel, PLB 753 (2016) 282 [arXiv:1508.07178]

Heupel, Eichmann, CF, PLB 718 (2021) 545 [arXiv:1206.5129]

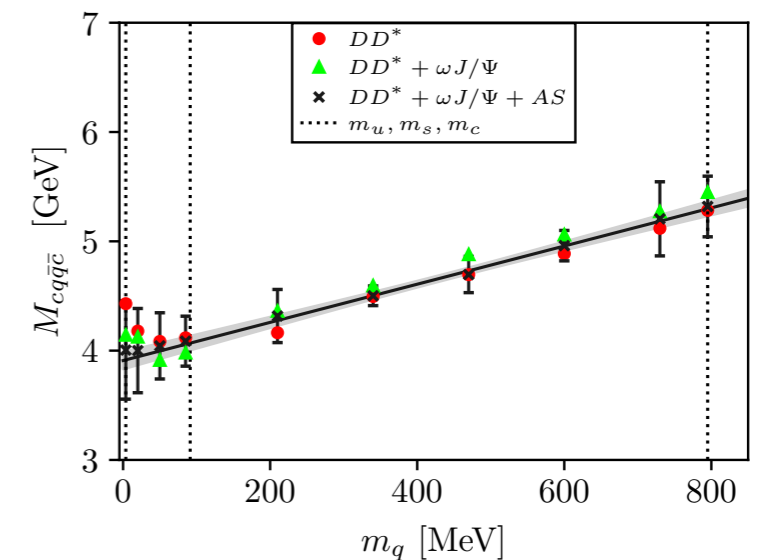
CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) 111077 [arXiv:2004.00415]



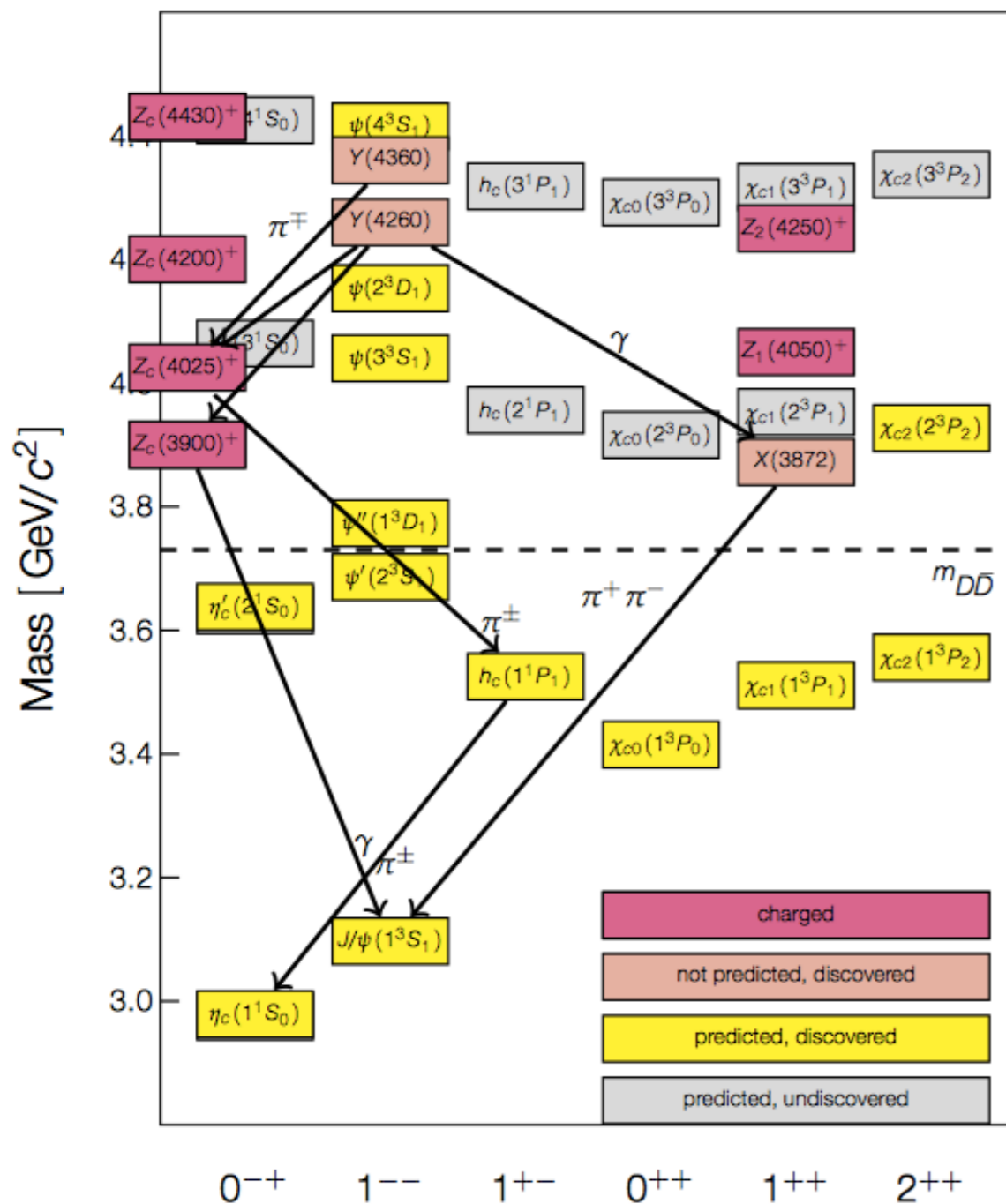
2. Heavy-light four-quark states: X(3872) and more...

Wallbott, Eichmann and CF, PRD100 (2019) no.1, 014033, [arXiv:1905.02615]

Wallbott, Eichmann and CF, PRD102 (2020) no.5, 051501, [arXiv:2003.12407]

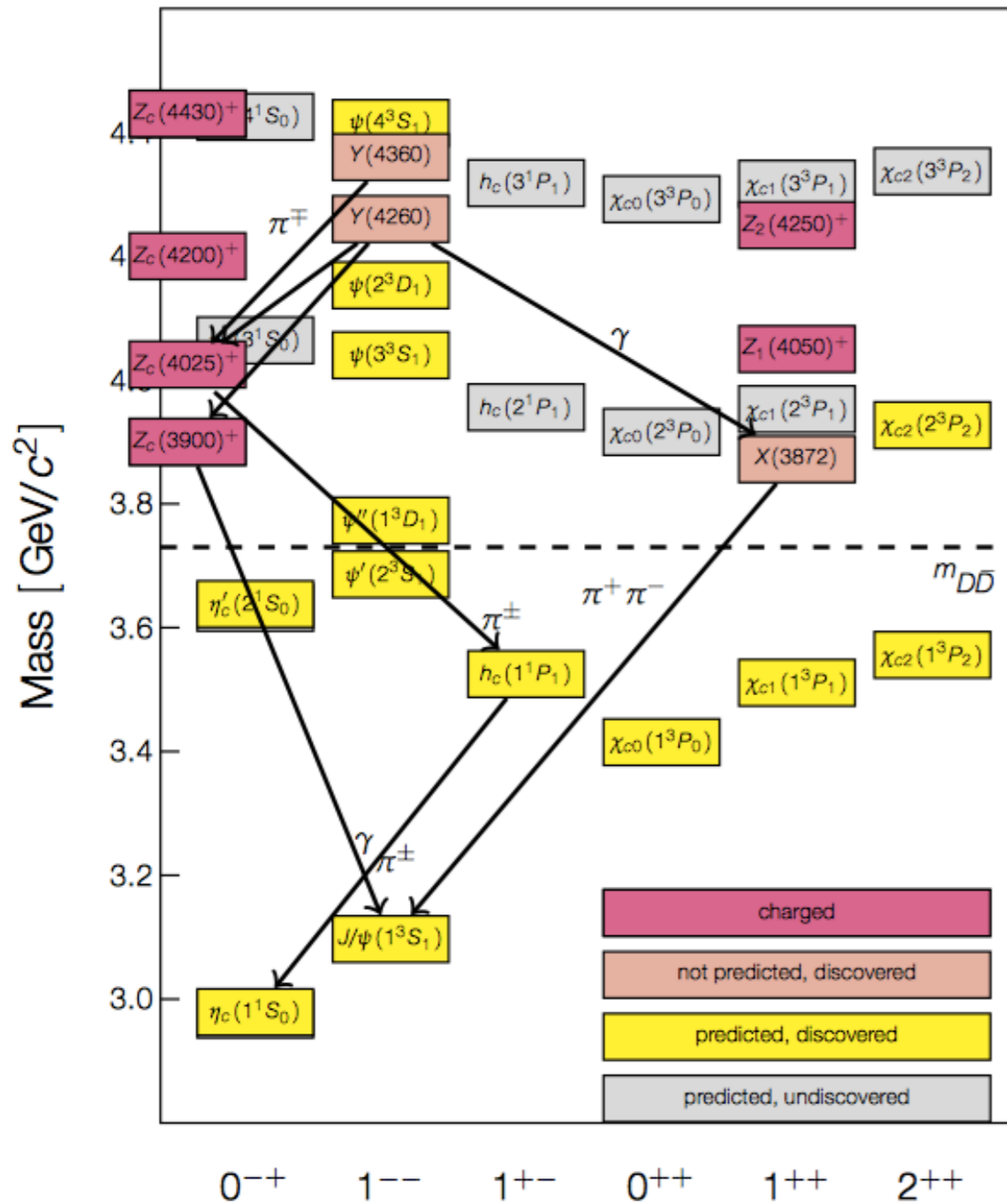


Tetraquark candidates with $cq\bar{q}\bar{c}$ -content



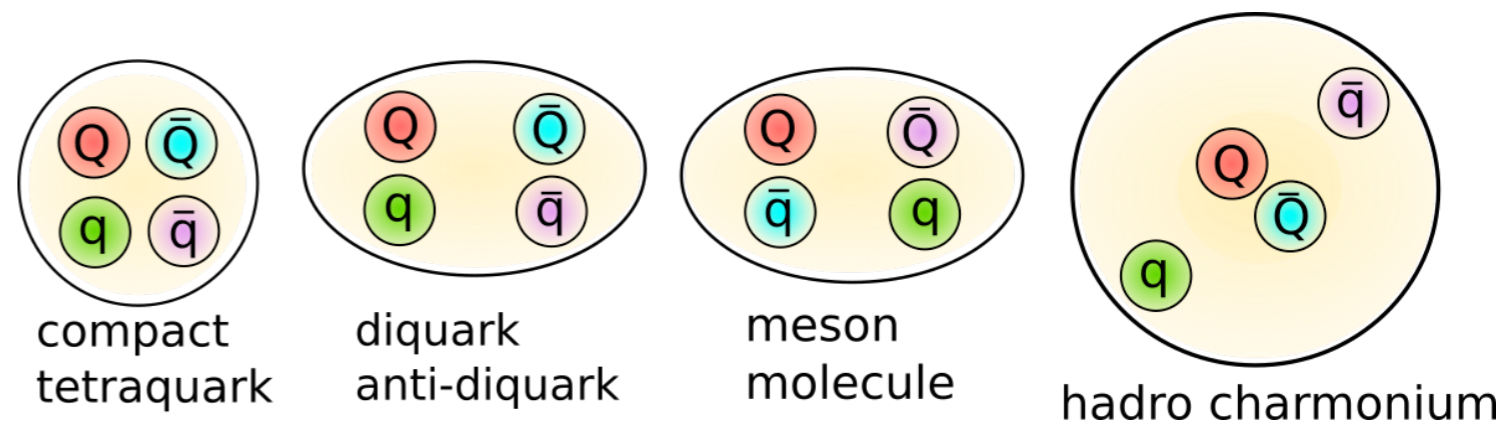
Many new unexpected states found: Belle, BABAR, BES, LHCb ...

Tetraquark candidates with $cq\bar{q}\bar{c}$ -content



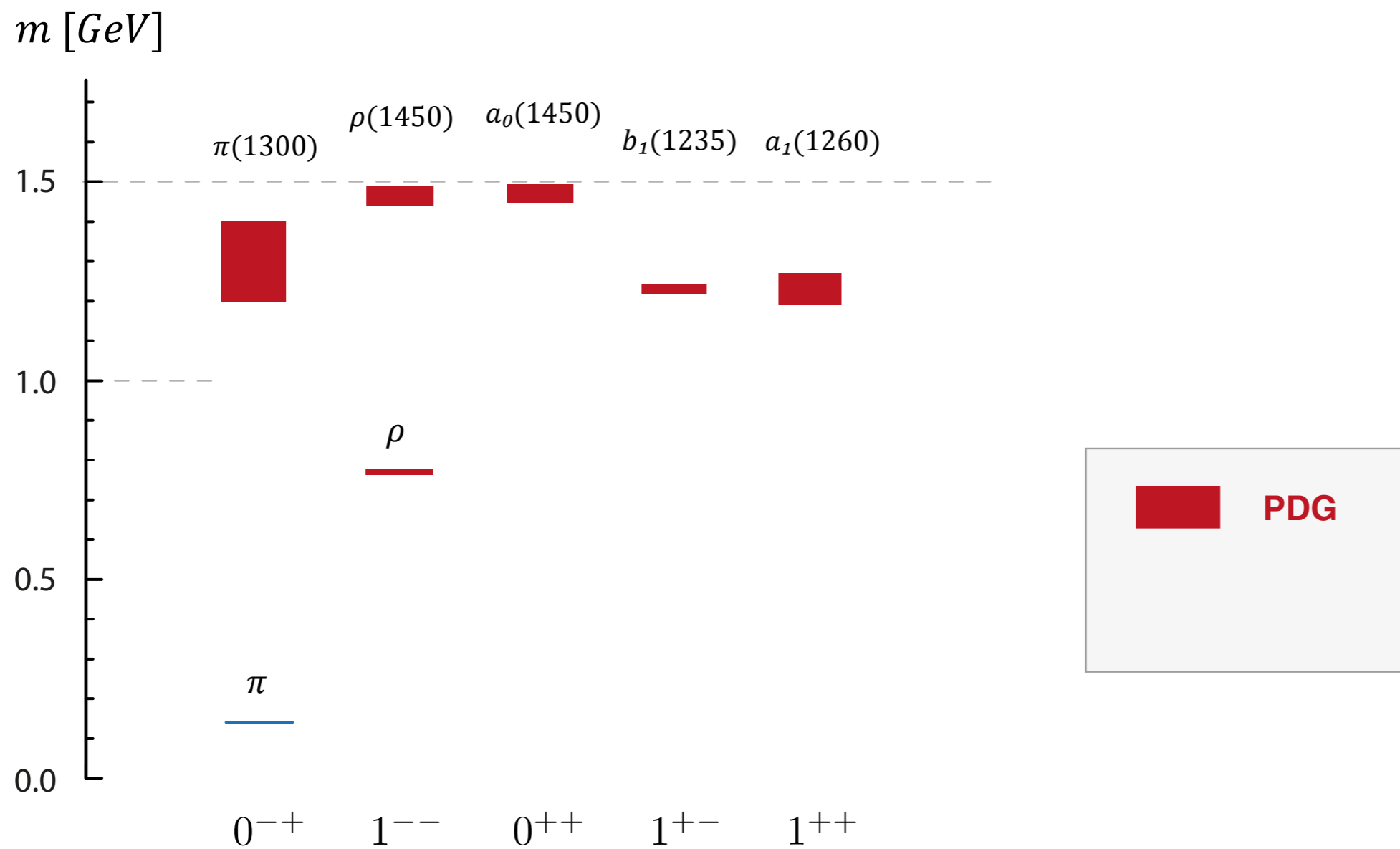
Many new unexpected states found: Belle, BABAR, BES, LHCb ...

Internal structure ??



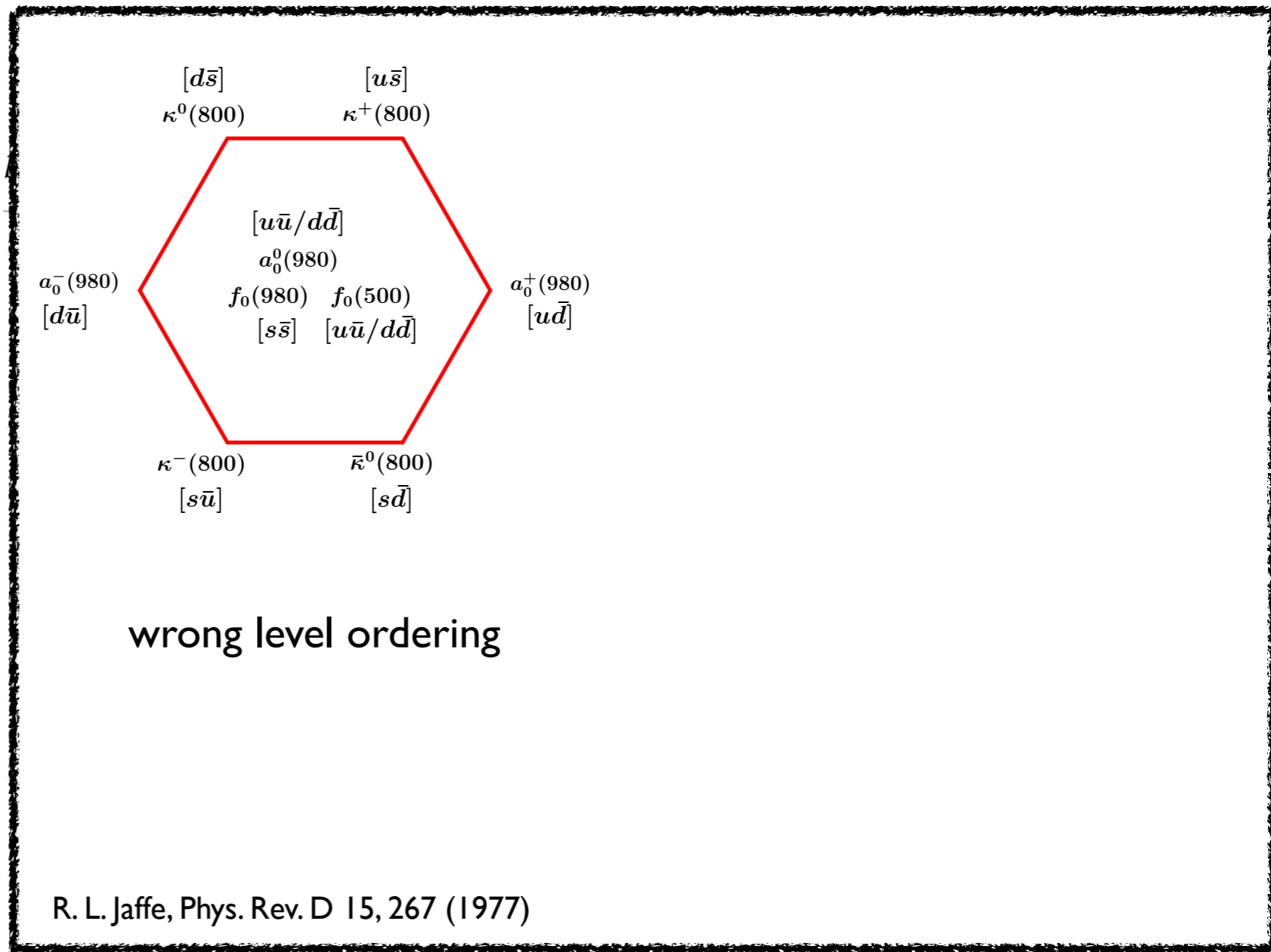
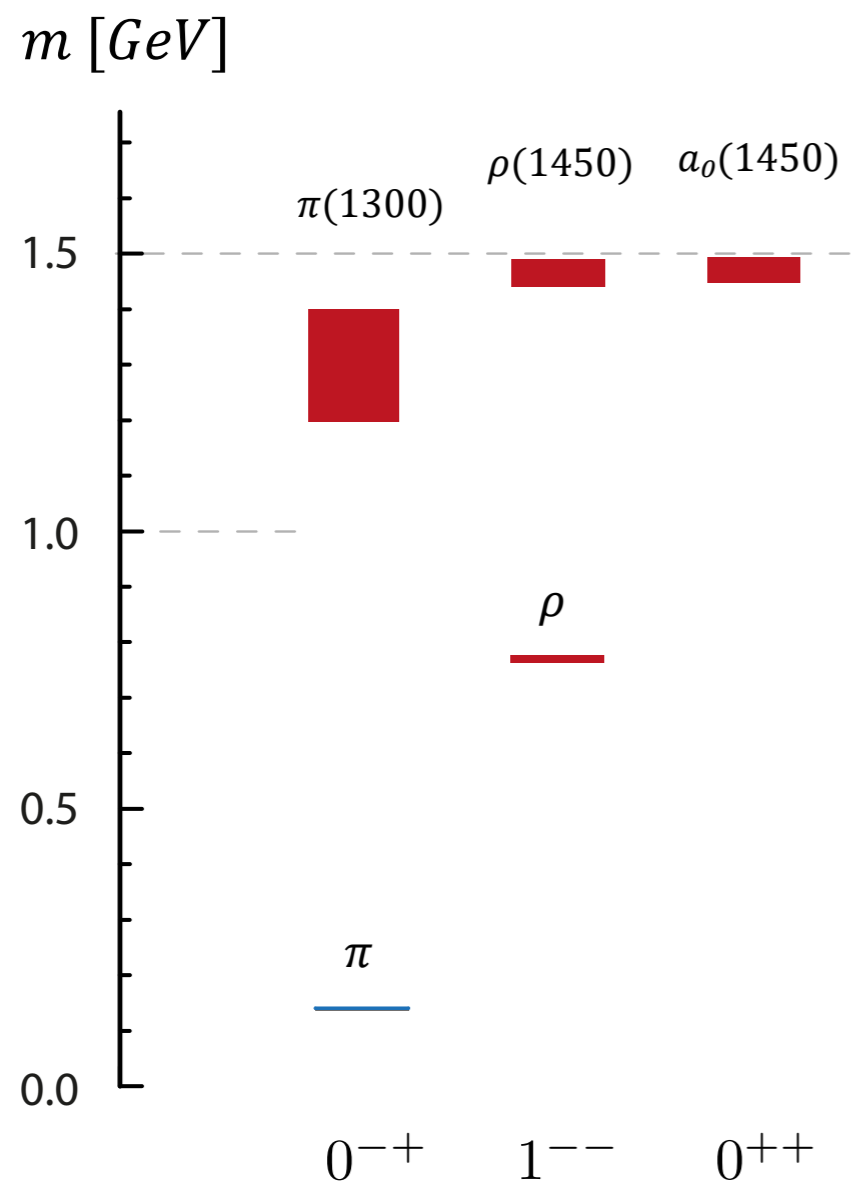
Related to details of underlying QCD forces between quarks and gluons

Conventional light meson spectrum



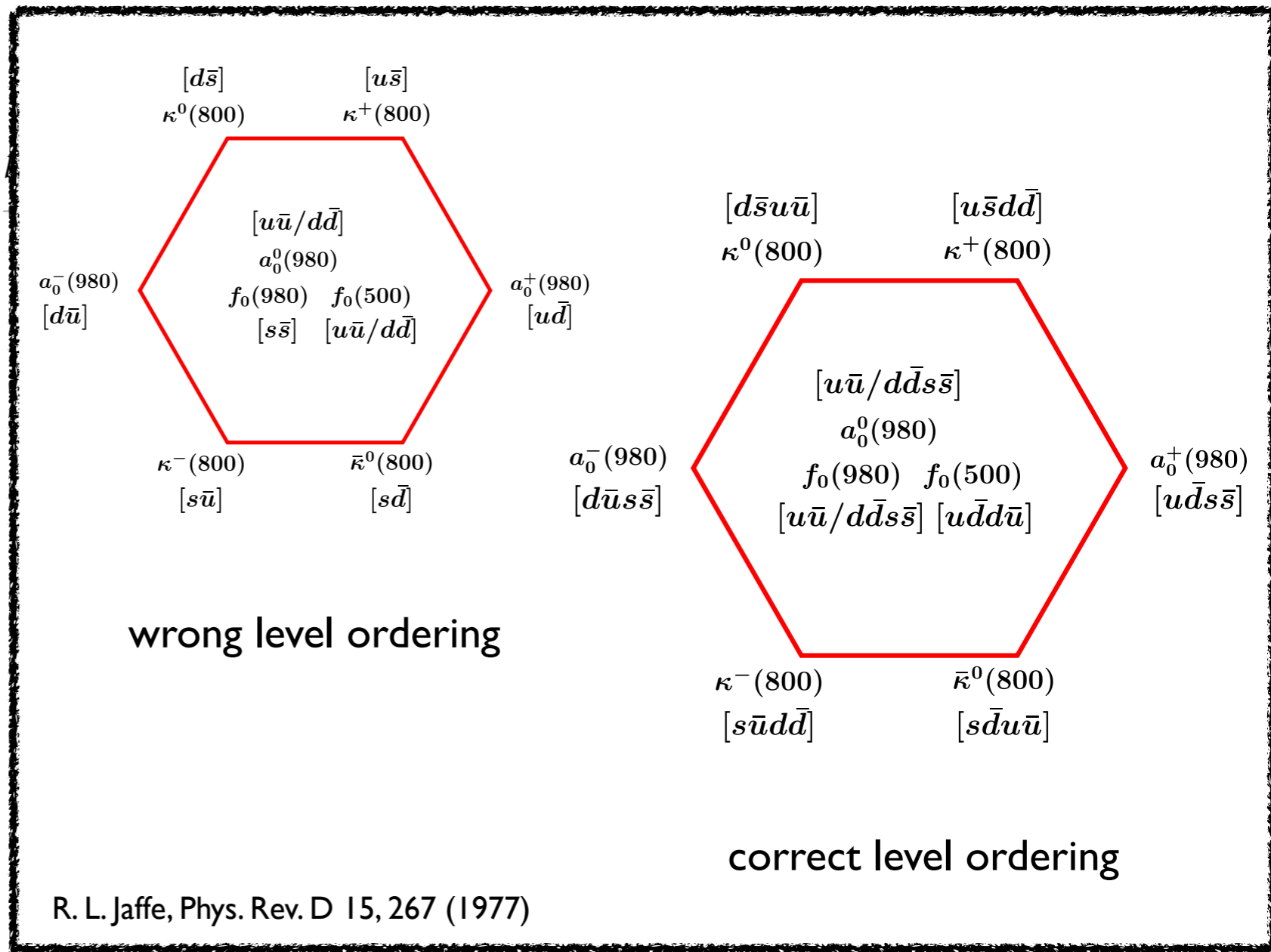
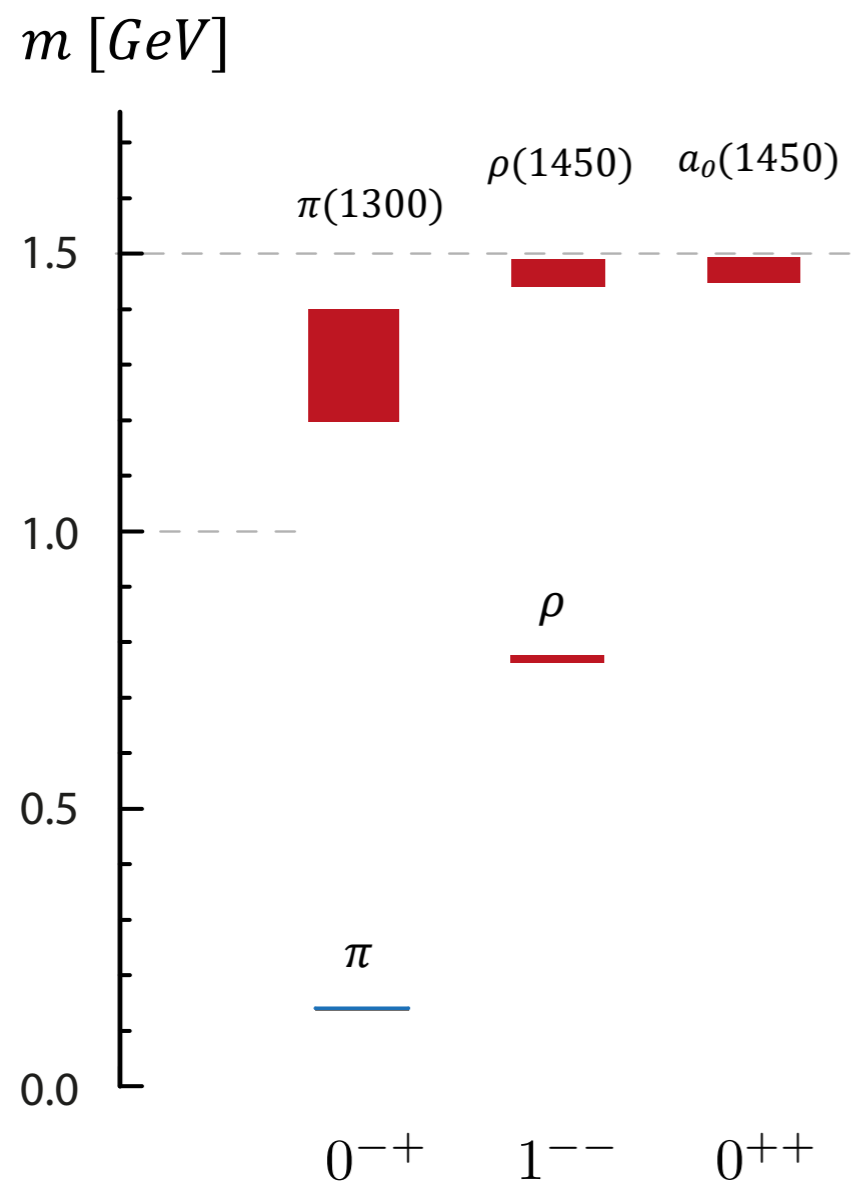
- ↑
- < 1 GeV: light nonet
 - > 1 GeV: 'heavy' nonet, glueball...

Conventional light meson spectrum



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Conventional light meson spectrum

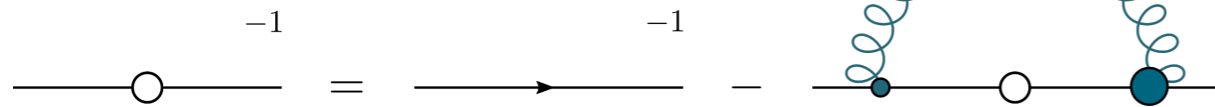


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- > 1 GeV: 'heavy' nonet, glueball...

Dyson-Schwinger equations - “3PI vs RL”

$$Z_{QCD} = \int \mathcal{D}[\Psi, A] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i\not{D} - m) \Psi - \frac{1}{4} (F_{\mu\nu}^a)^2 \right) \right\}$$

propagators

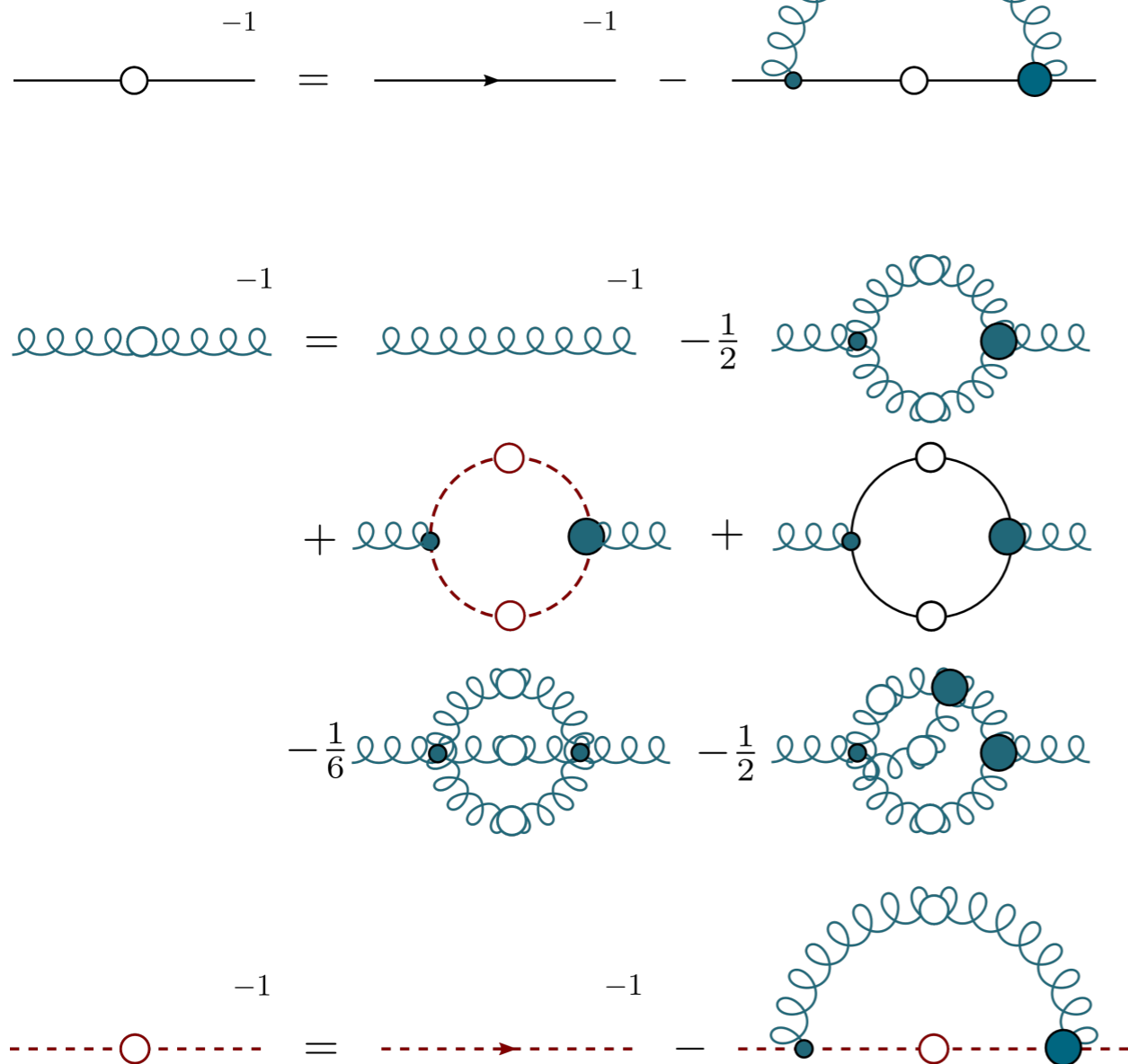


CF,Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

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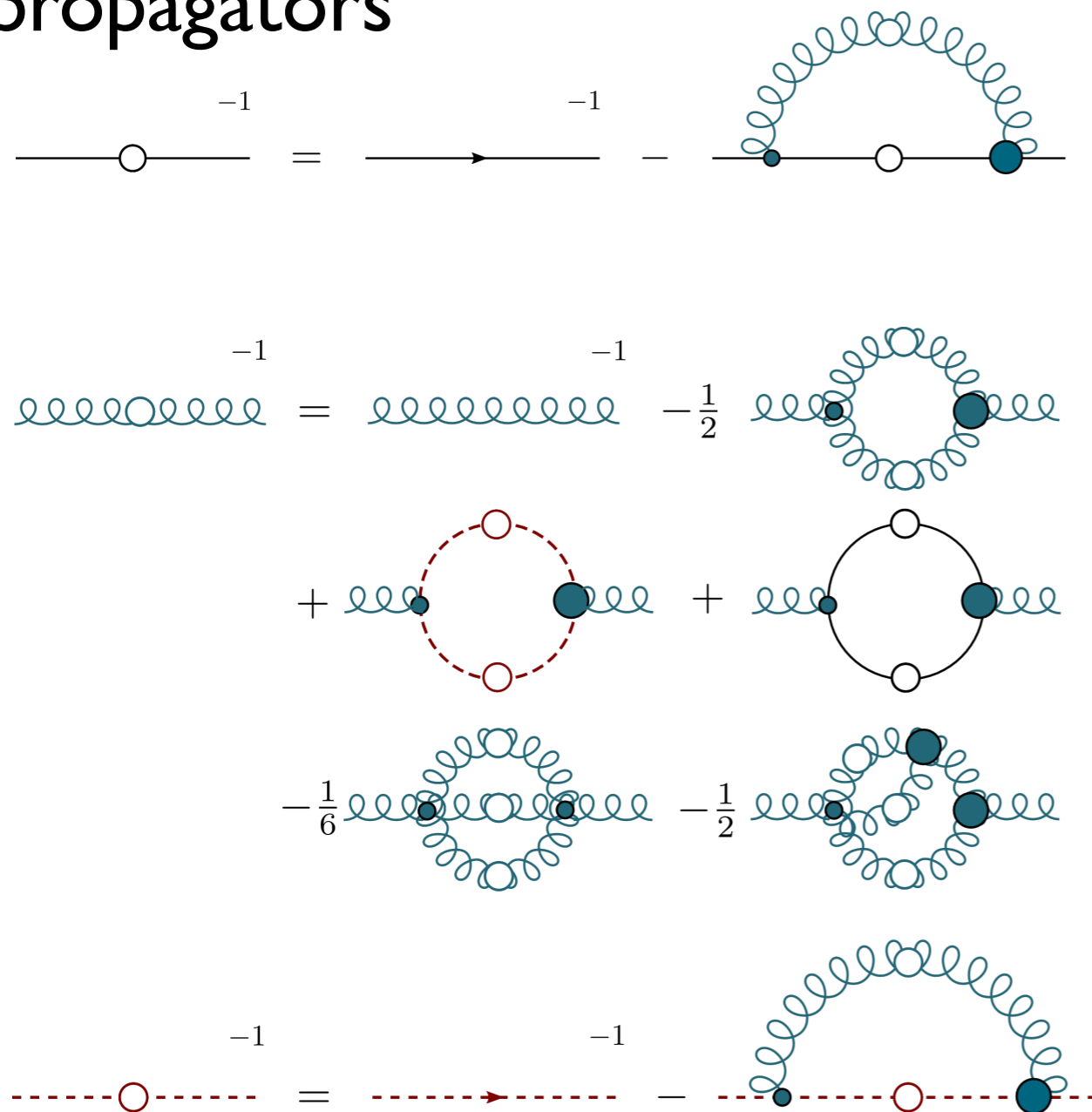


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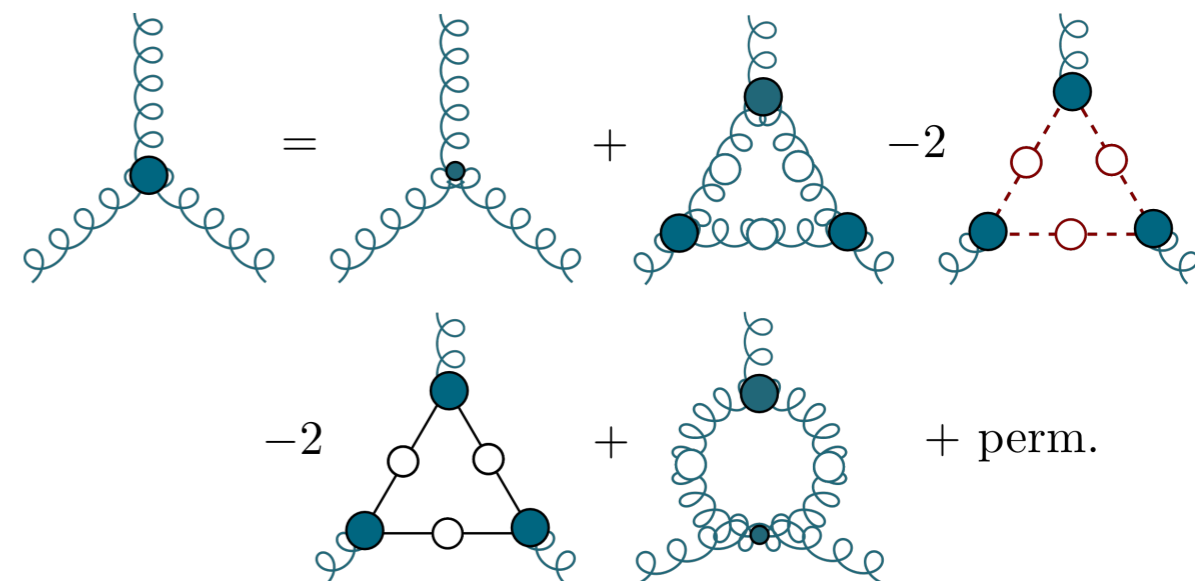
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vertices

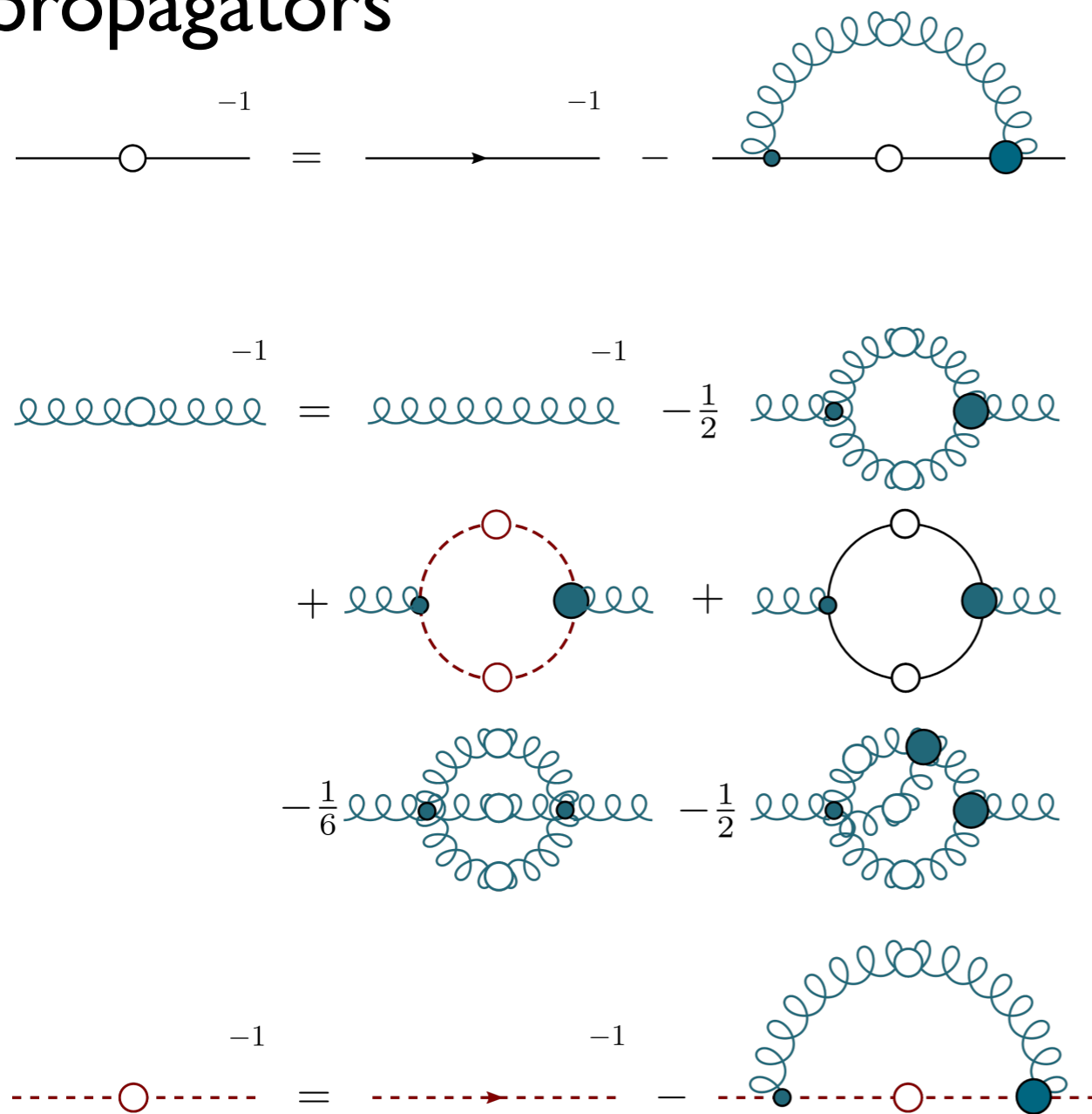


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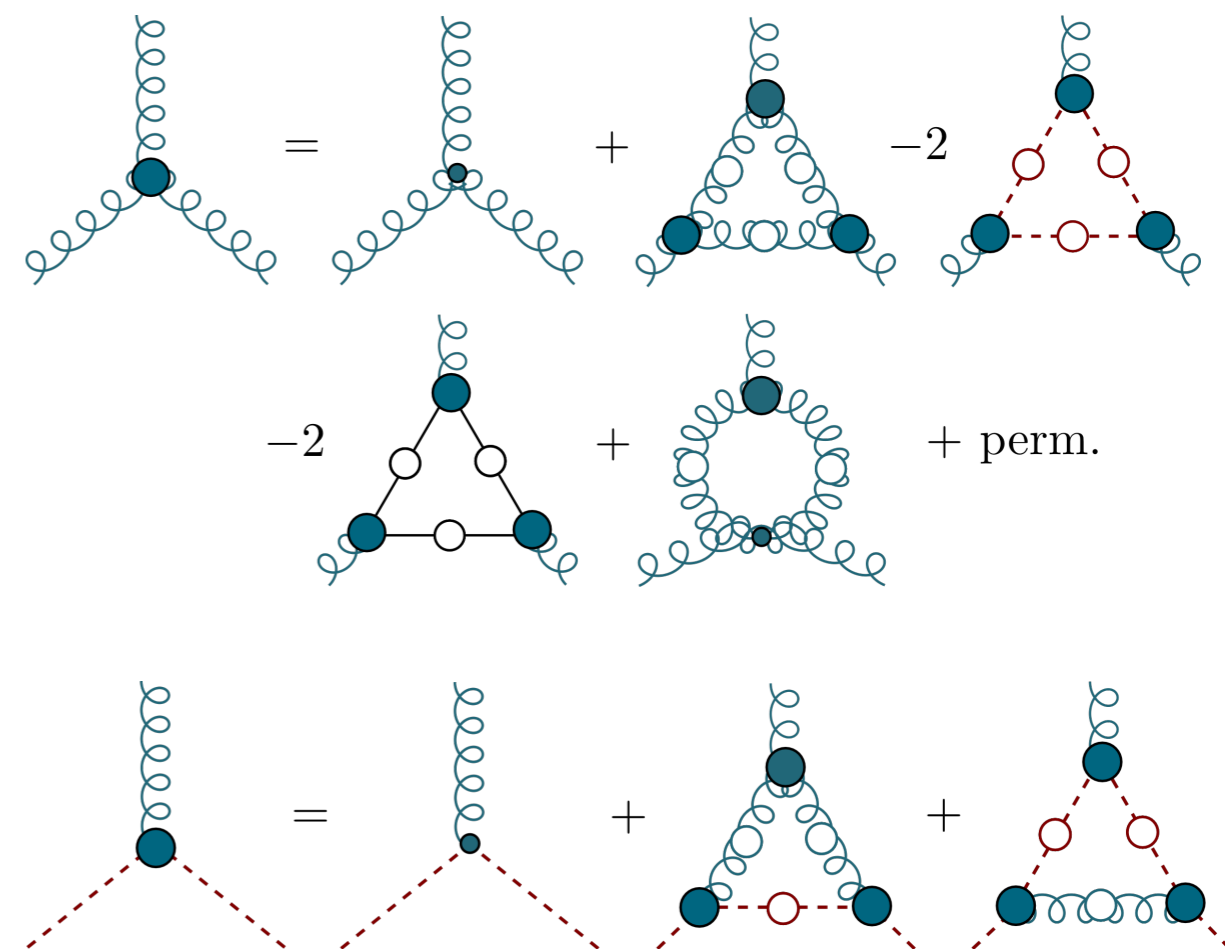
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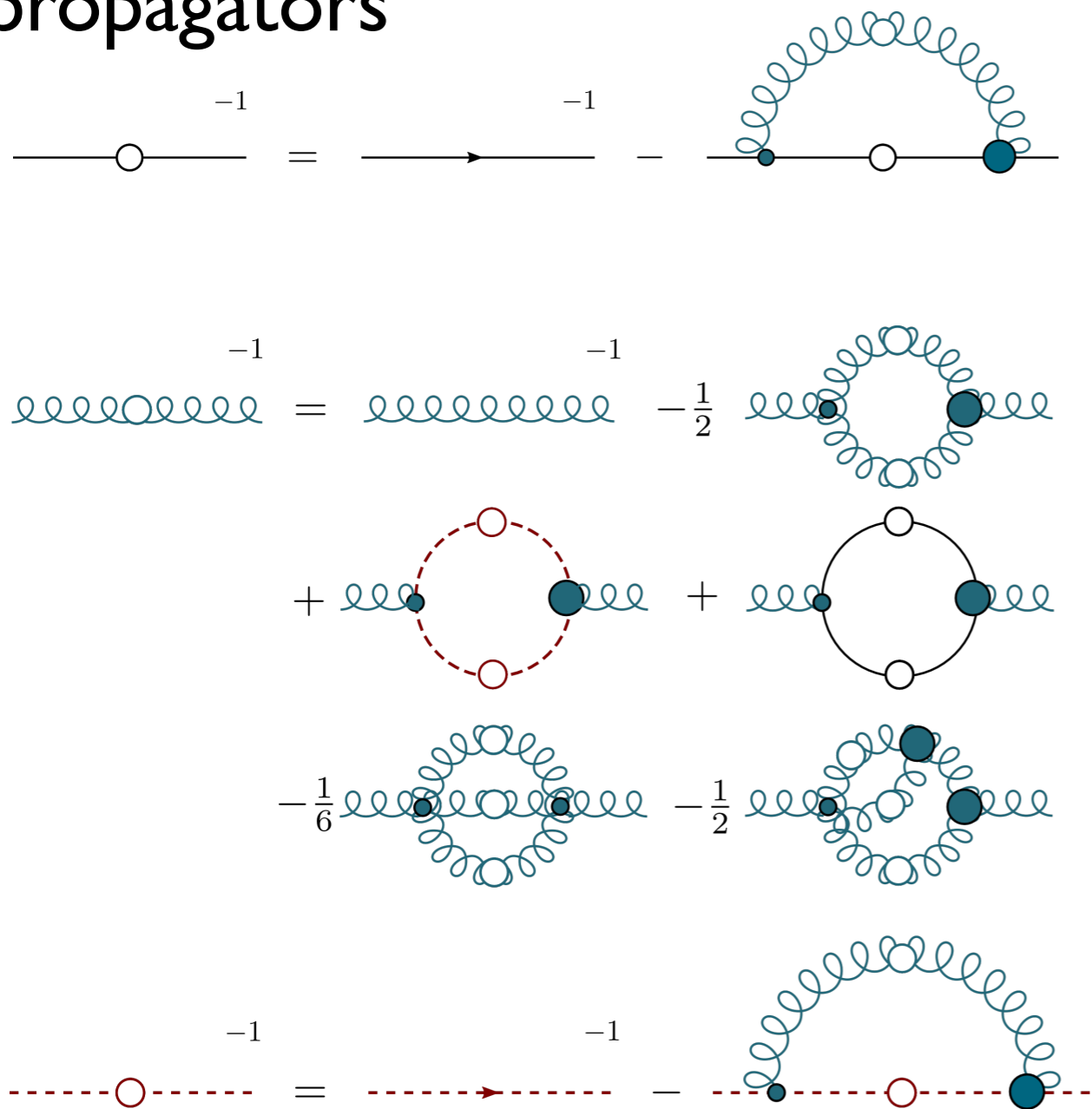


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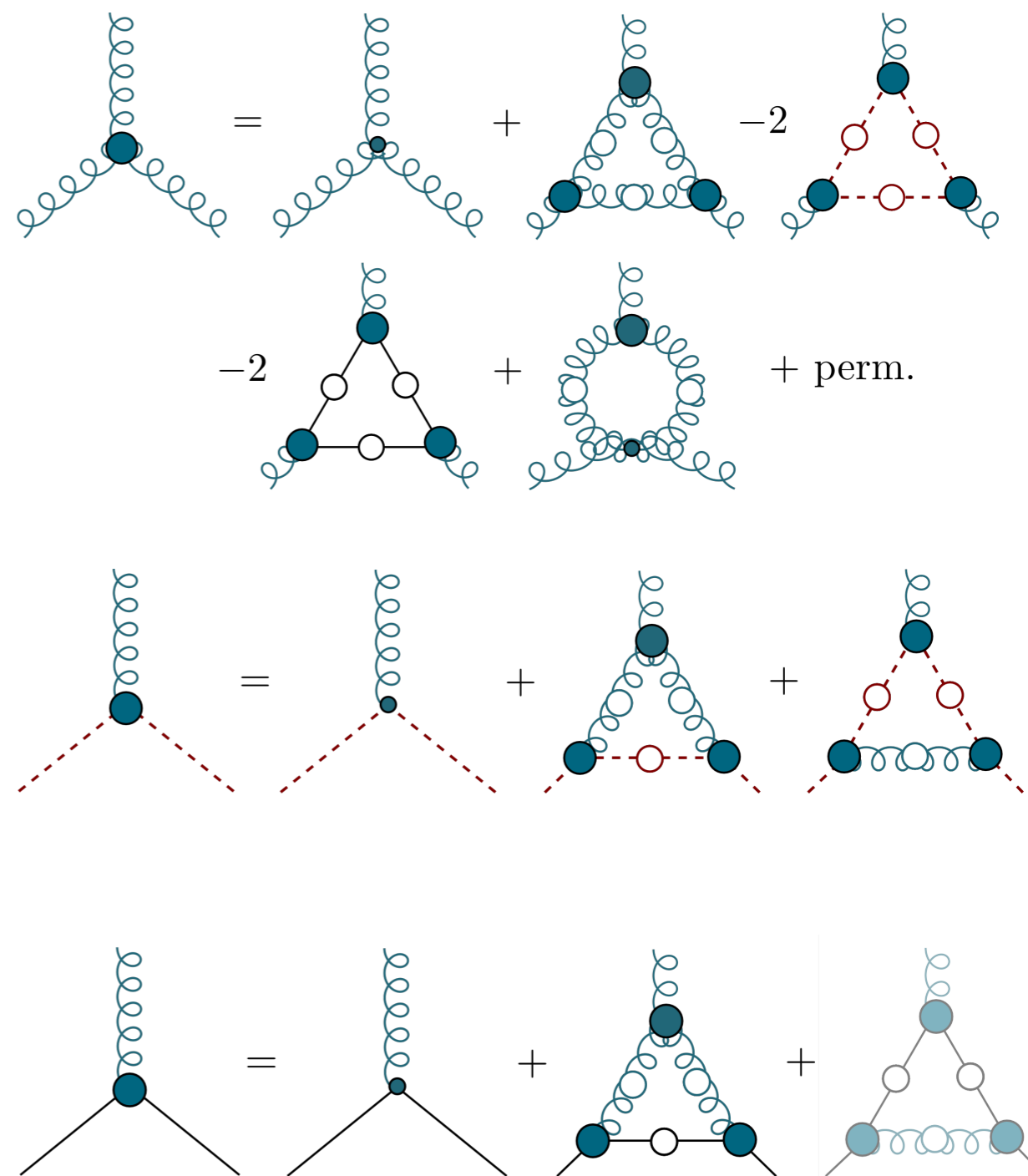
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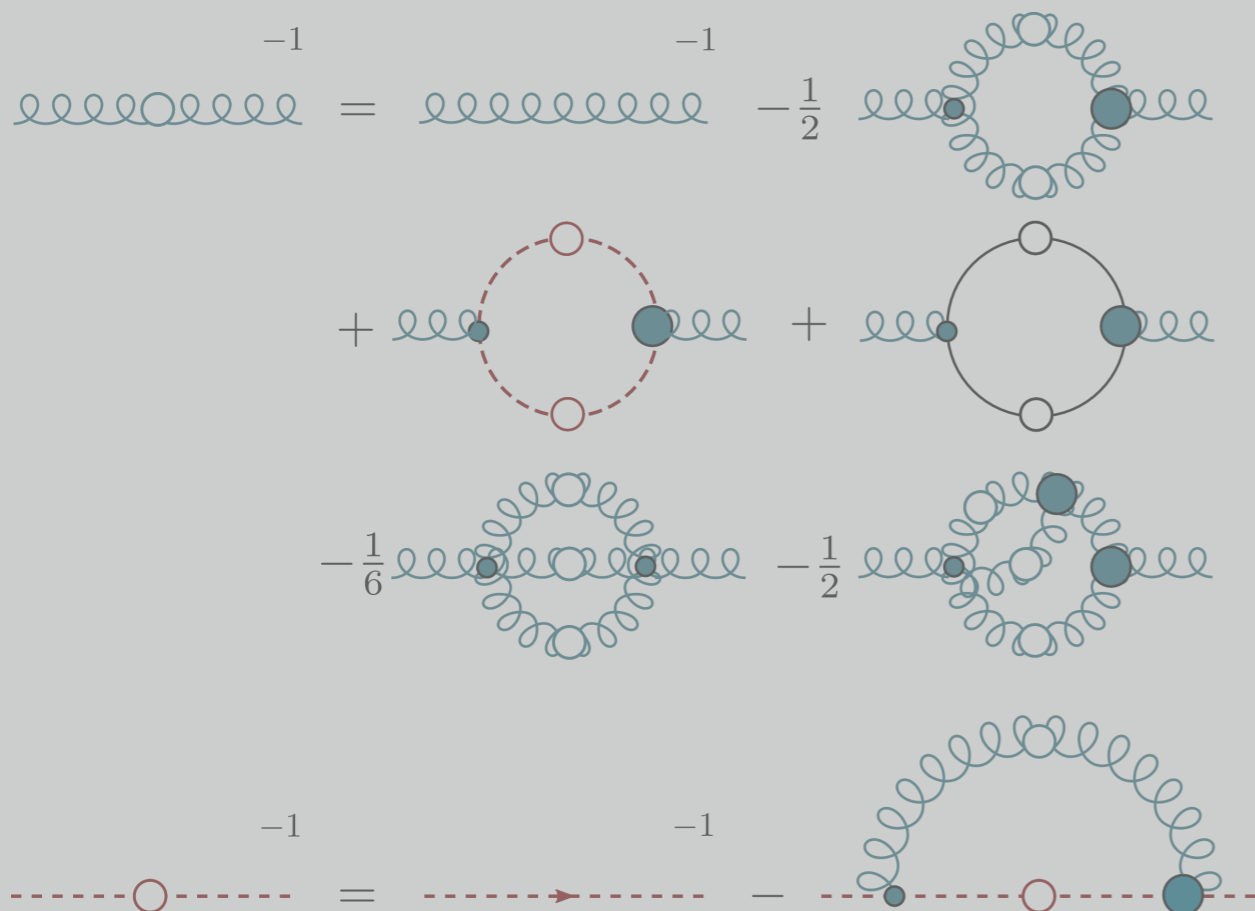
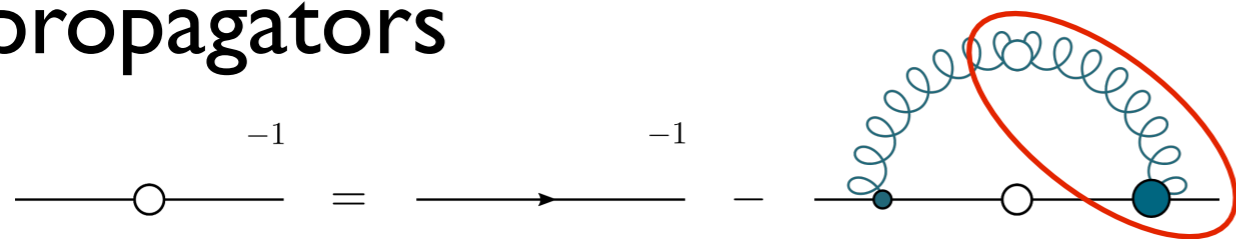


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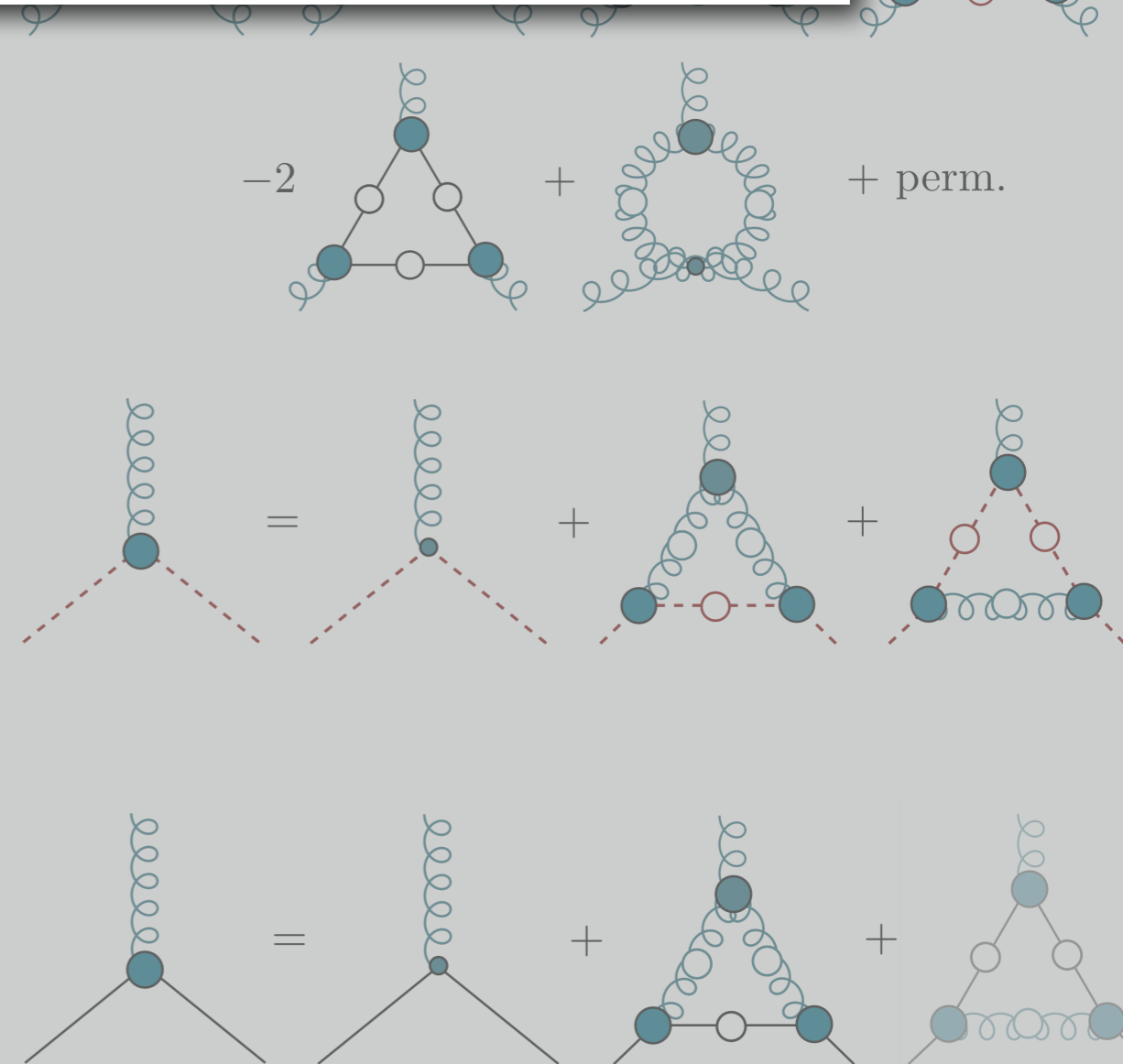
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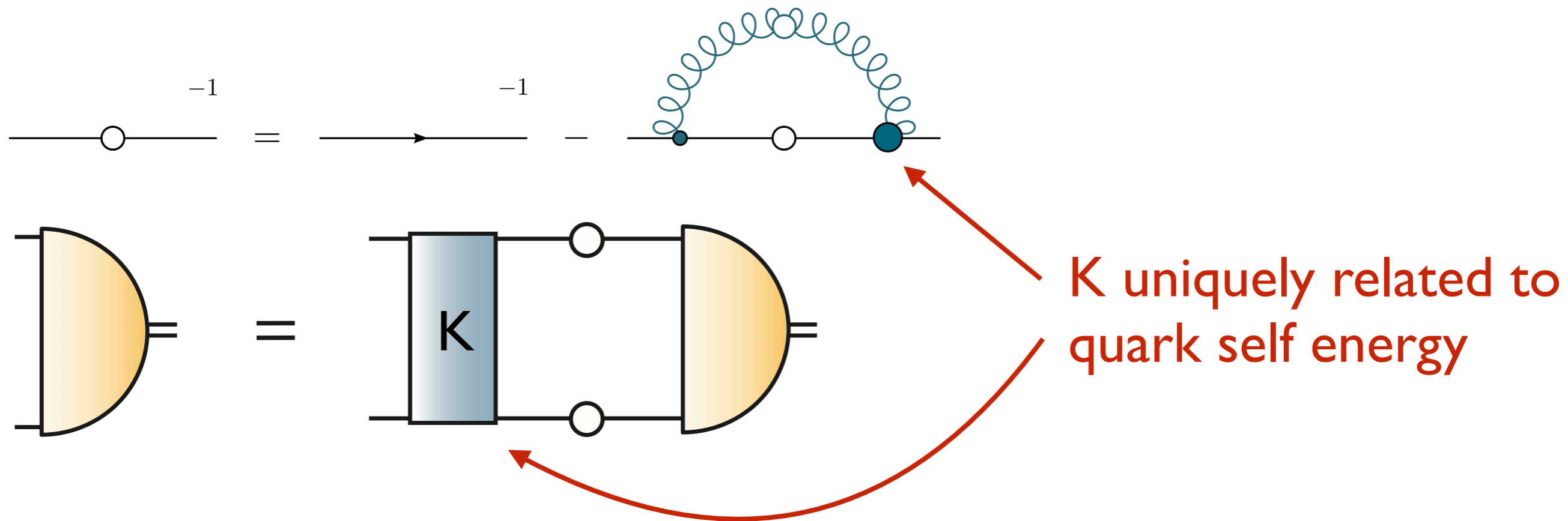
vertices

“rainbow-ladder” (RL) :
model for gluon+vertex



CF,Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

Mesons as quark-antiquark states

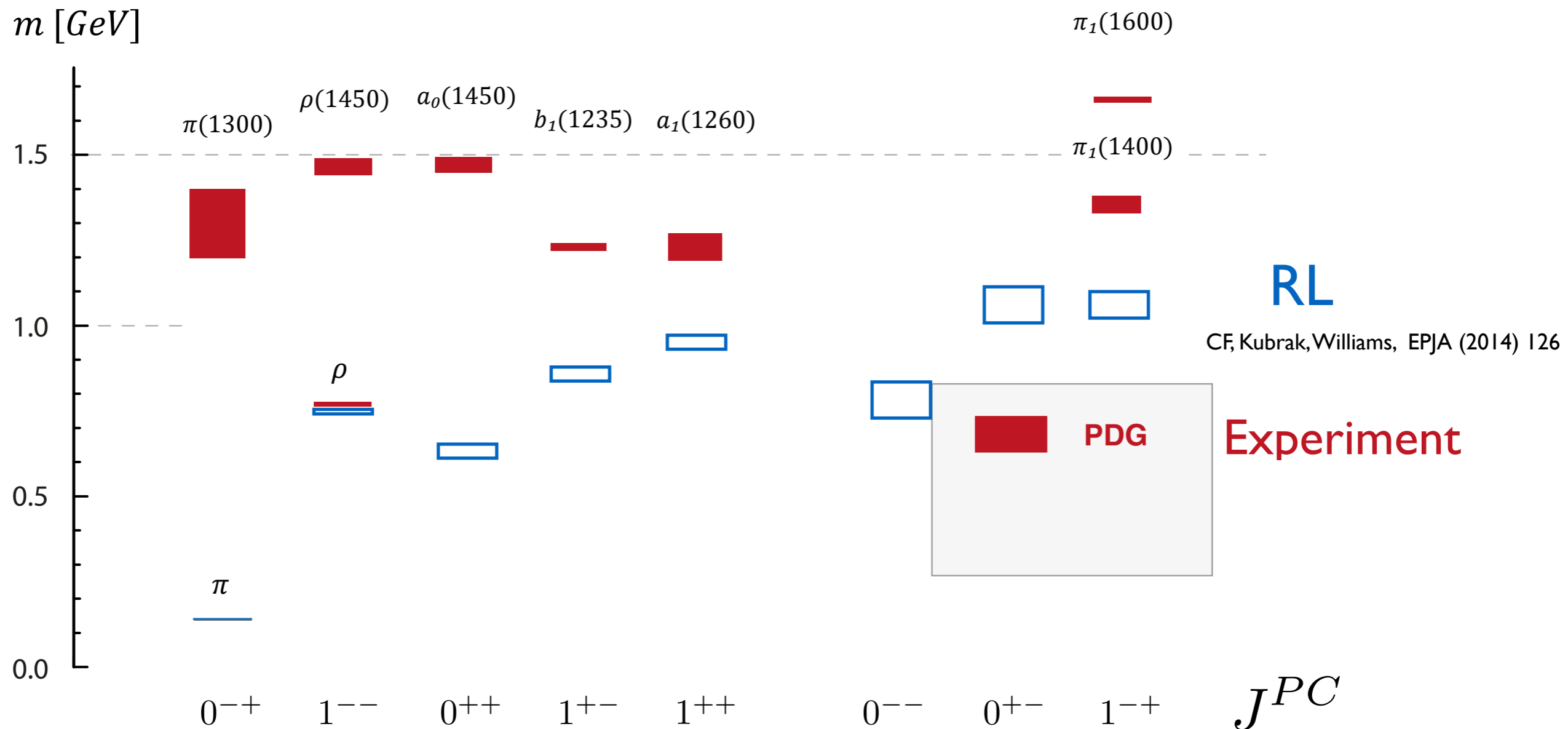


→ Pion is bound state **and** Goldstone boson

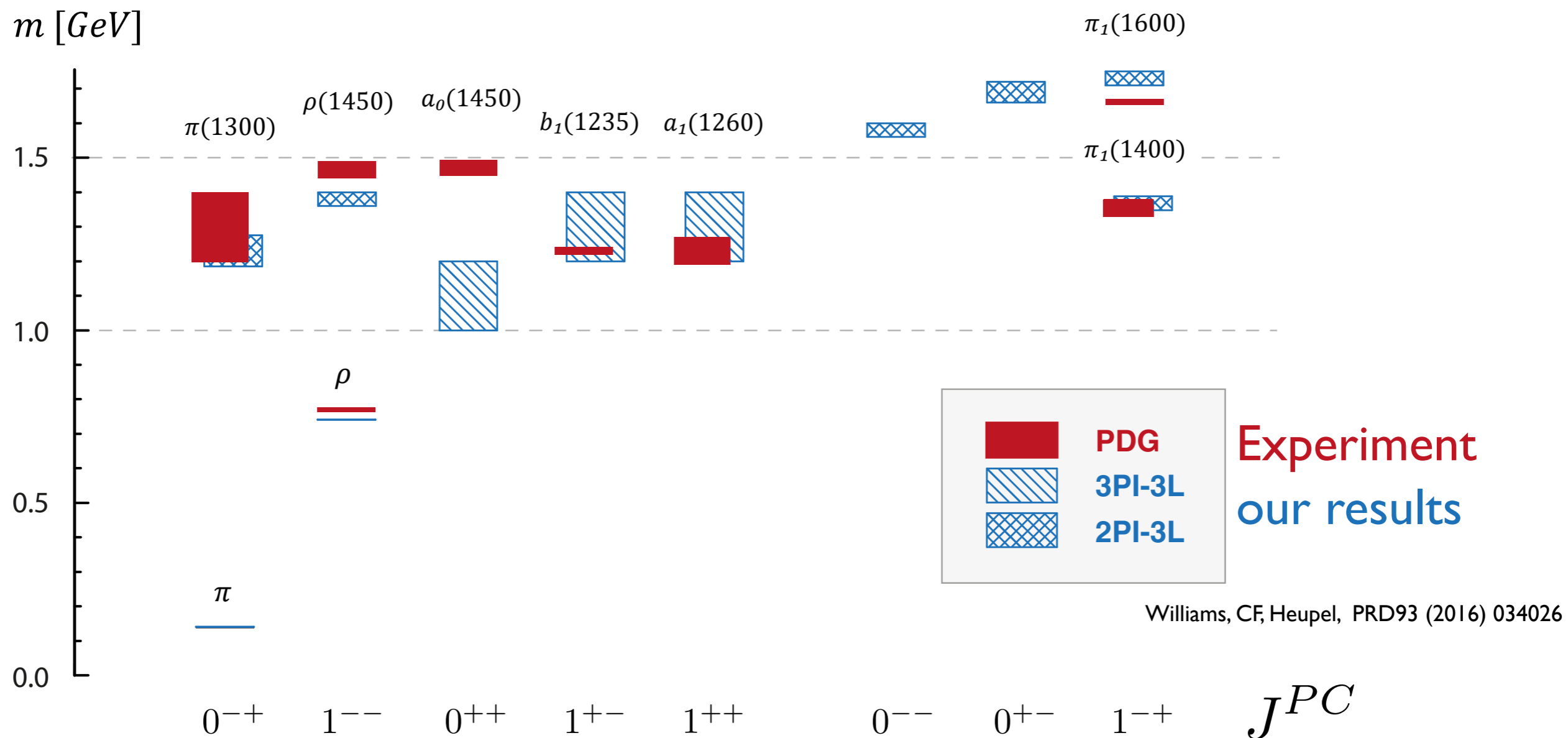
Maris, Roberts, Tandy, PLB 420 (1998) 267

- Determine gauge invariant spectrum from underlying gauge dependent quark/gluon dynamics

Light meson spectrum - full 3PI-calculation

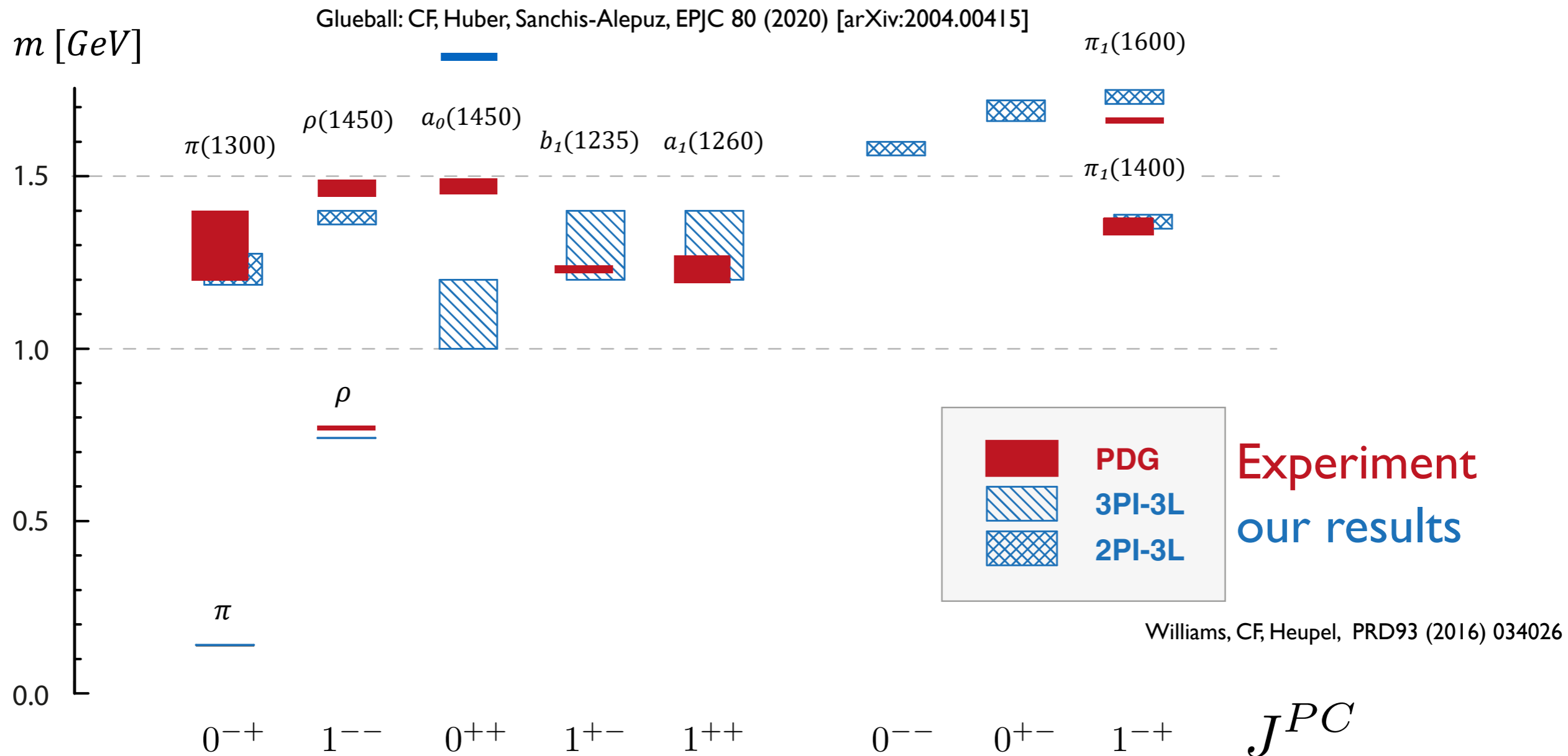


Light meson spectrum - full 3PI-calculation



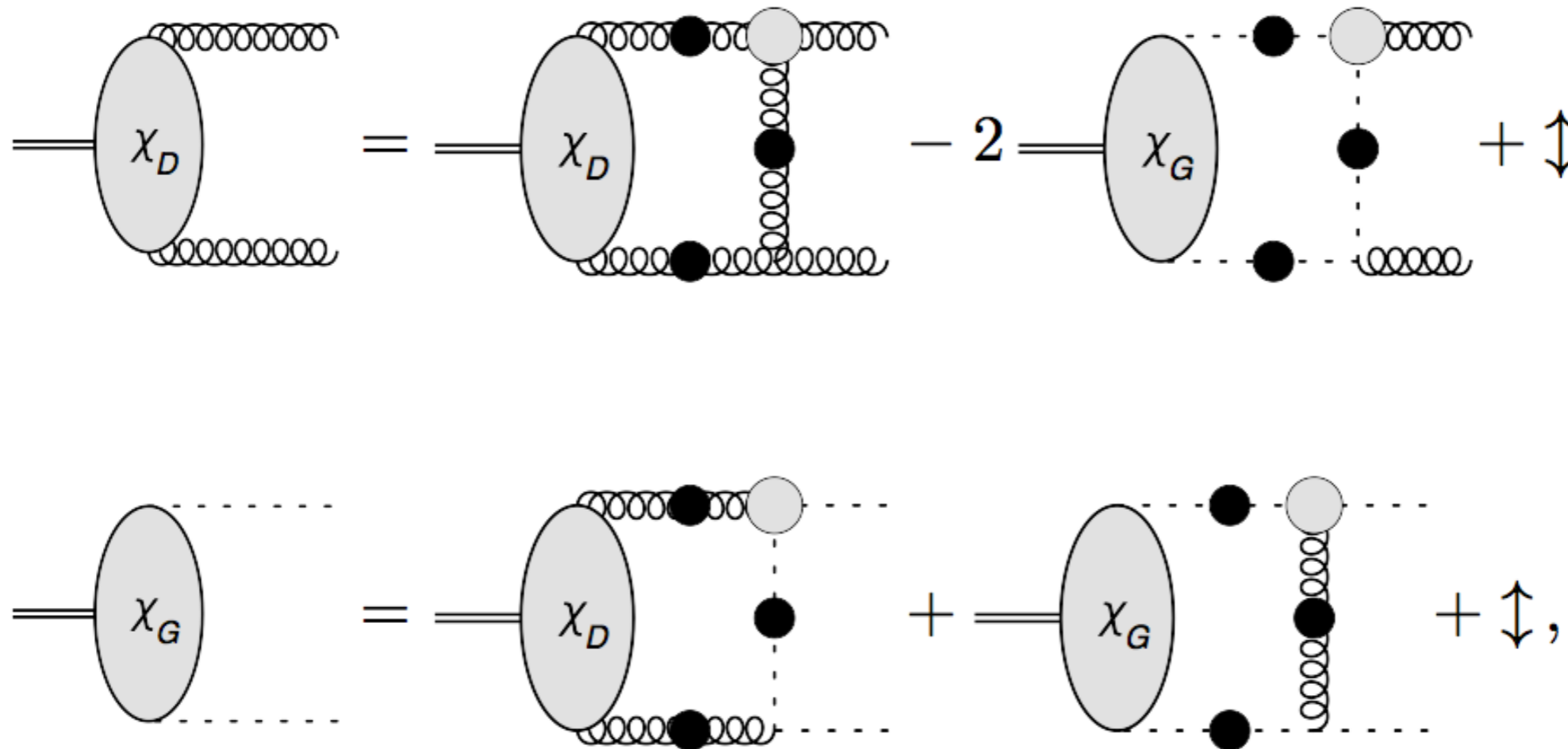
- good agreement with experiment in most channels
- special channels:
 pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons
 scalar 0^{++} : complicated channel...

Light meson spectrum - full 3PI-calculation



- good agreement with experiment in most channels
- special channels:
 - pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons
 - scalar 0^{++} : complicated channel...

Glueballs from DSE/BSEs



- Mixing of two-gluon amplitudes with ghost-antighost

- exploratory: simple models

Meyers, Swanson, PRD 87 (2013) 3, 036009

Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

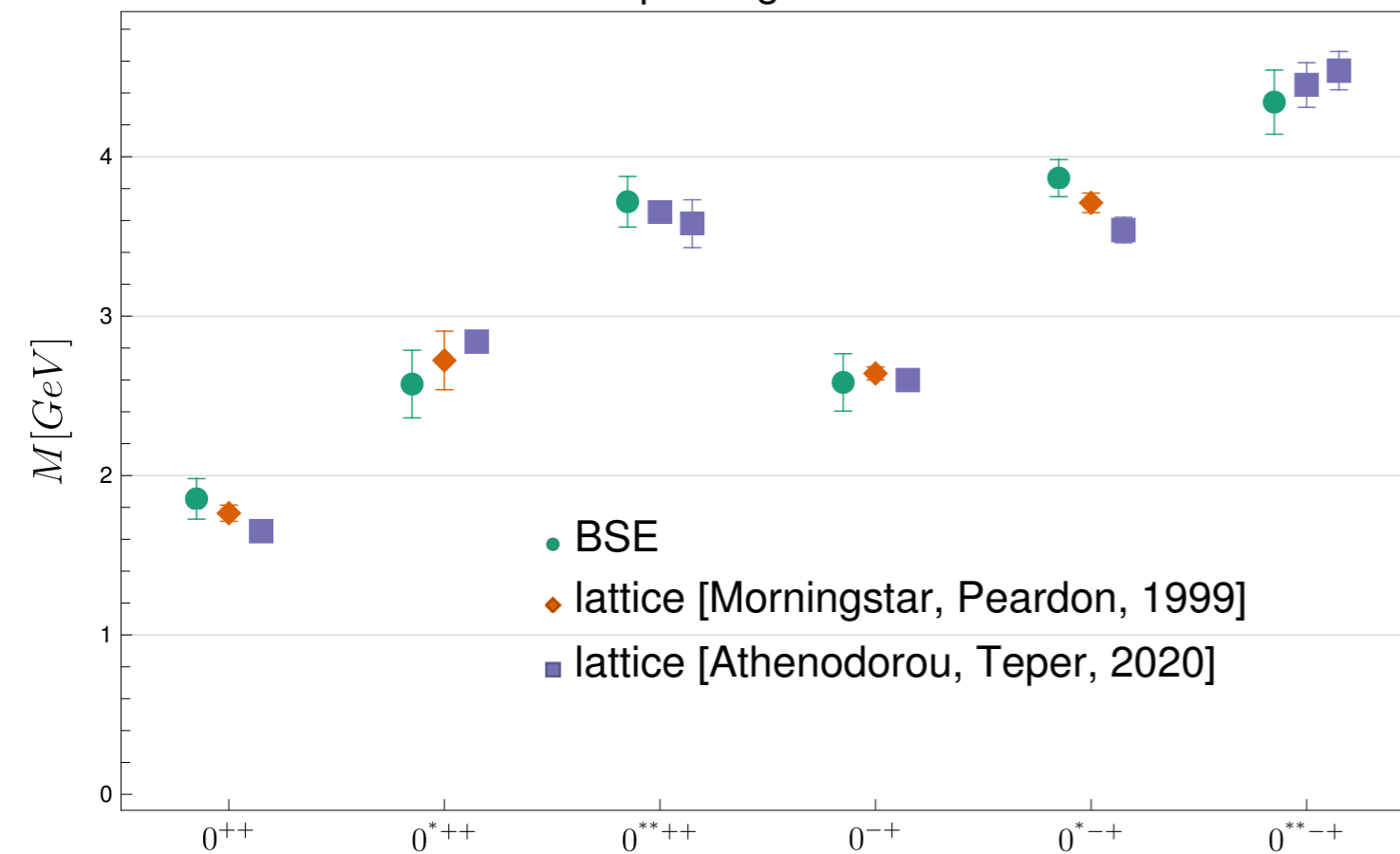
- new: high quality input from 3PI truncation

Huber, PRD 101 (2020) 114009

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020)

Glueballs: results

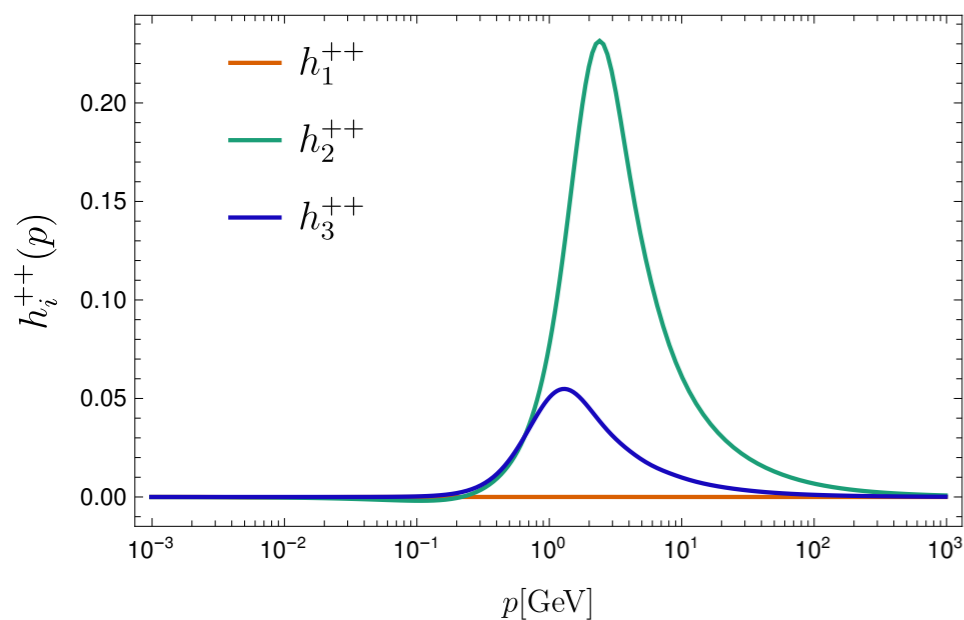
Spin-0 glueballs



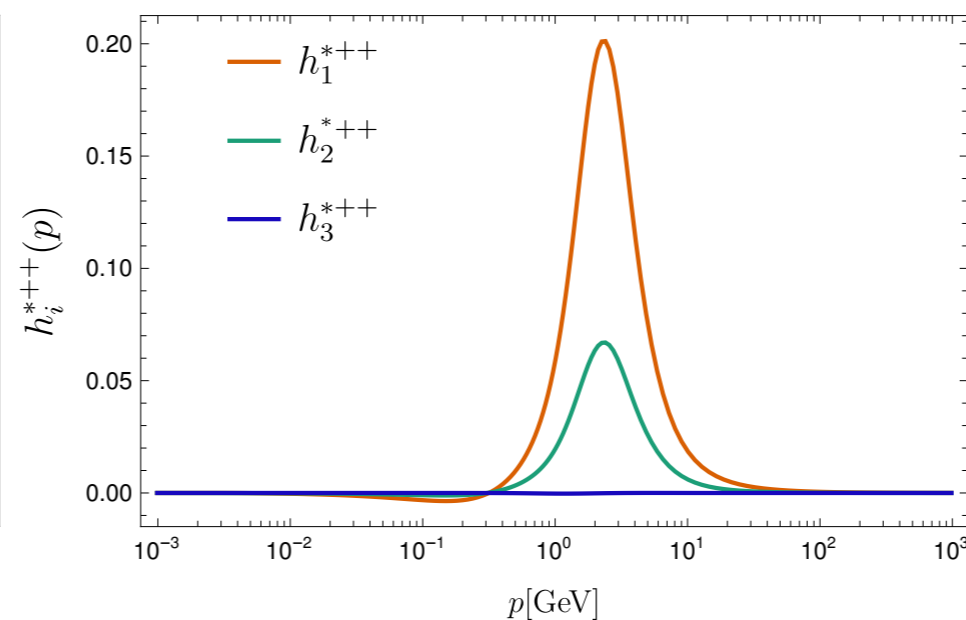
● spectrum:
very good agreement
lattice vs. DSE/BSE

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]

Amplitudes 0^{++}



Amplitudes 0^{*++}

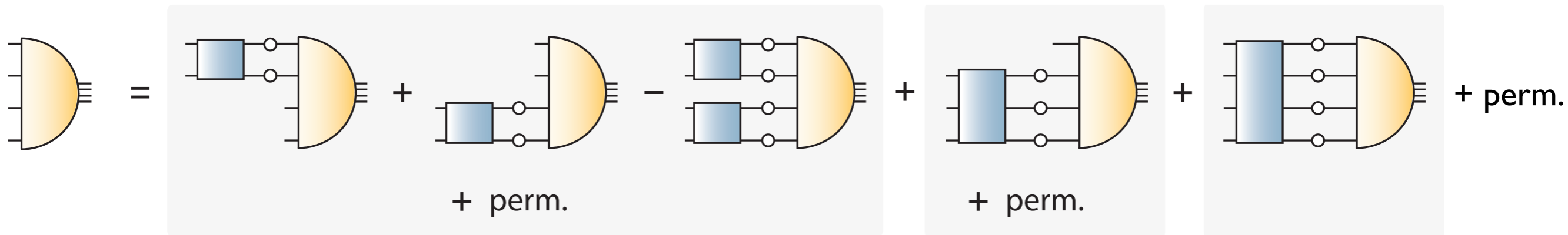


● excited states:
different internal
structure

Tetraquarks from the four-body equation

Kvinikhidze, Khvedelidze, Theor. Math. Phys. 90 (1992)
Heupel, Eichmann, CF, PLB 718 (2012) 545-549
Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Kvinikhidze, Blankleider, arXiv:2102.09558.

Exact equation:



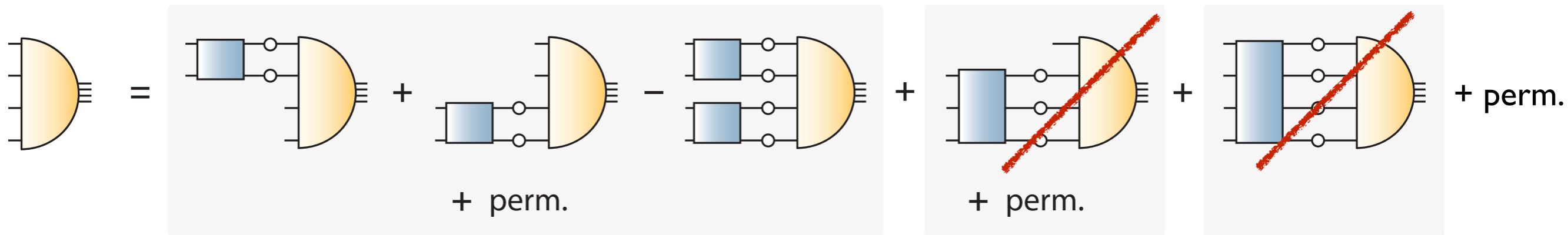
Two-body interactions

Three- and four-body interactions

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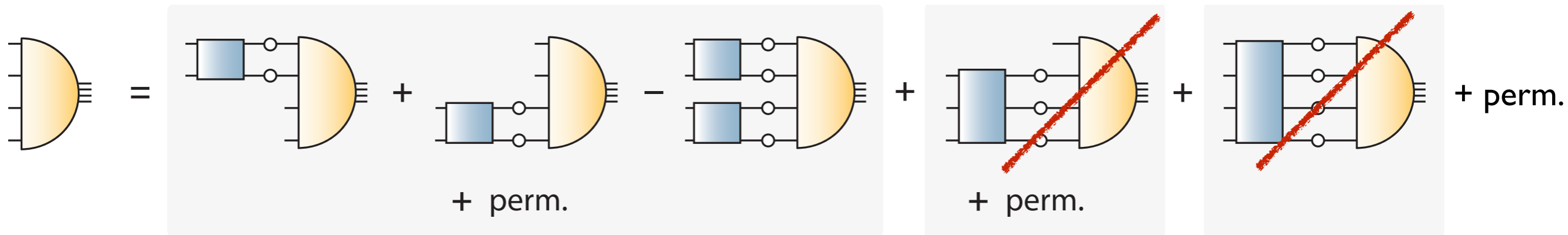
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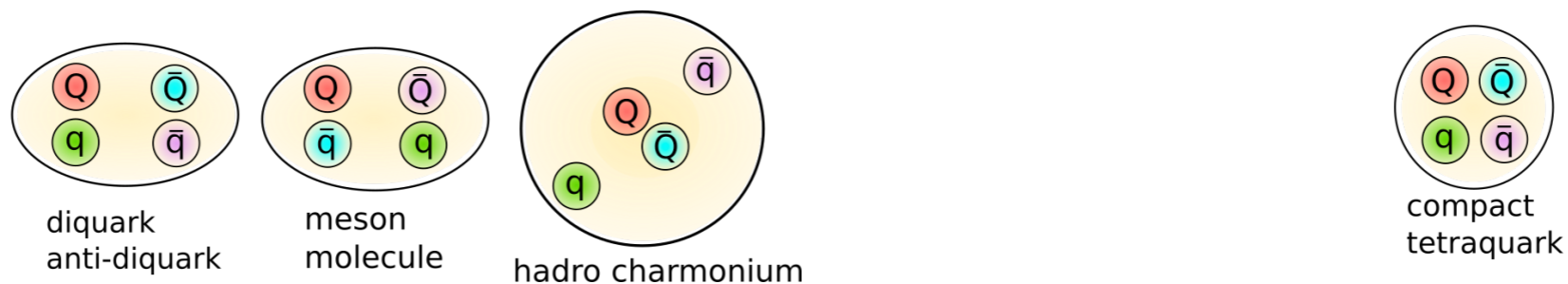
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Two-body interactions

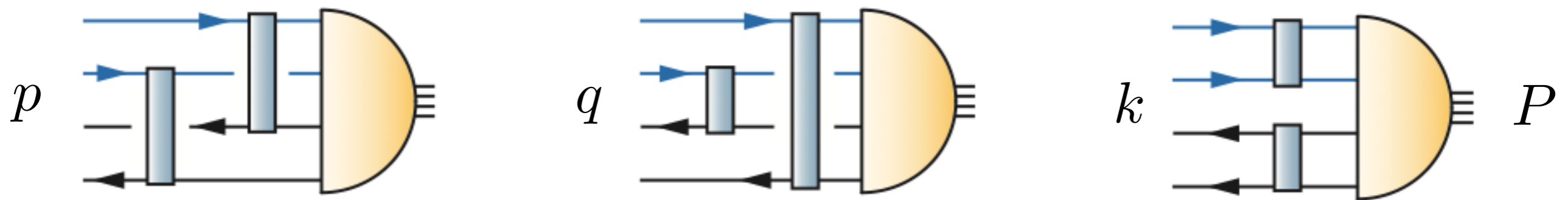
Three- and four-body interactions



- Two-body interactions: allow for **internal clustering**
- rainbow-ladder approximation ok

Structure of four-quark amplitude I

Scalar four-quark state:

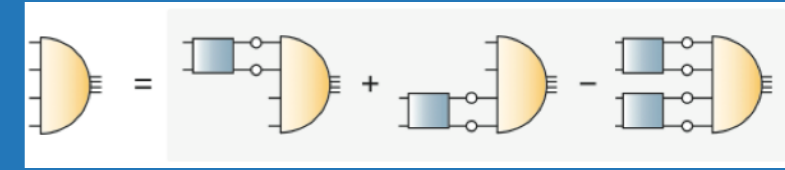


$$\Gamma(P, p, q, k) = \sum_i f_i(s_1, \dots, s_9) \times \tau_i(P, p, q, k) \times color \times flavor$$

9 Lorentz scalars 256 tensor 3 ⊗ $\bar{3}$, 6 ⊗ $\bar{6}$ or
(built from P,p,q,k) structures 1 ⊗ 1, 8 ⊗ 8

- reduce # tensor structures guided by physics:
→ ~20 tensor structures

Structure of four-quark amplitude II

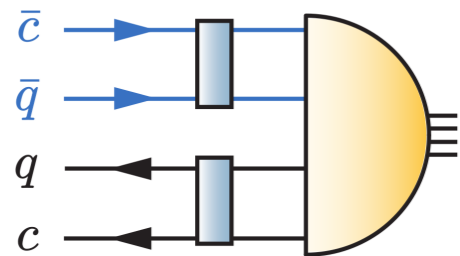


● **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$

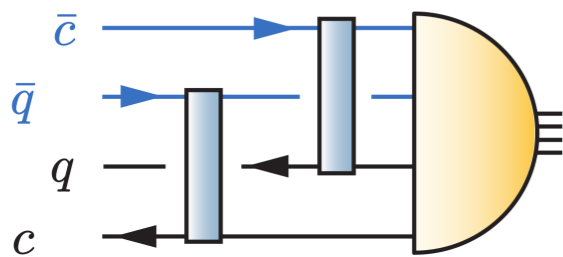
● **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$

$\mathcal{D}_2 \sim q^2 - p^2$

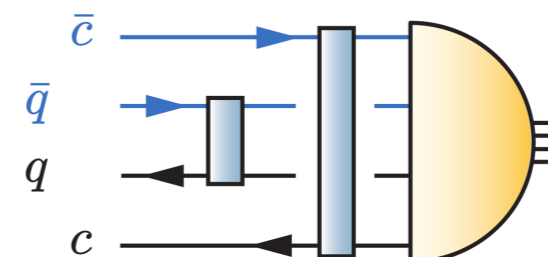
● **Two triplets: mostly irrelevant**



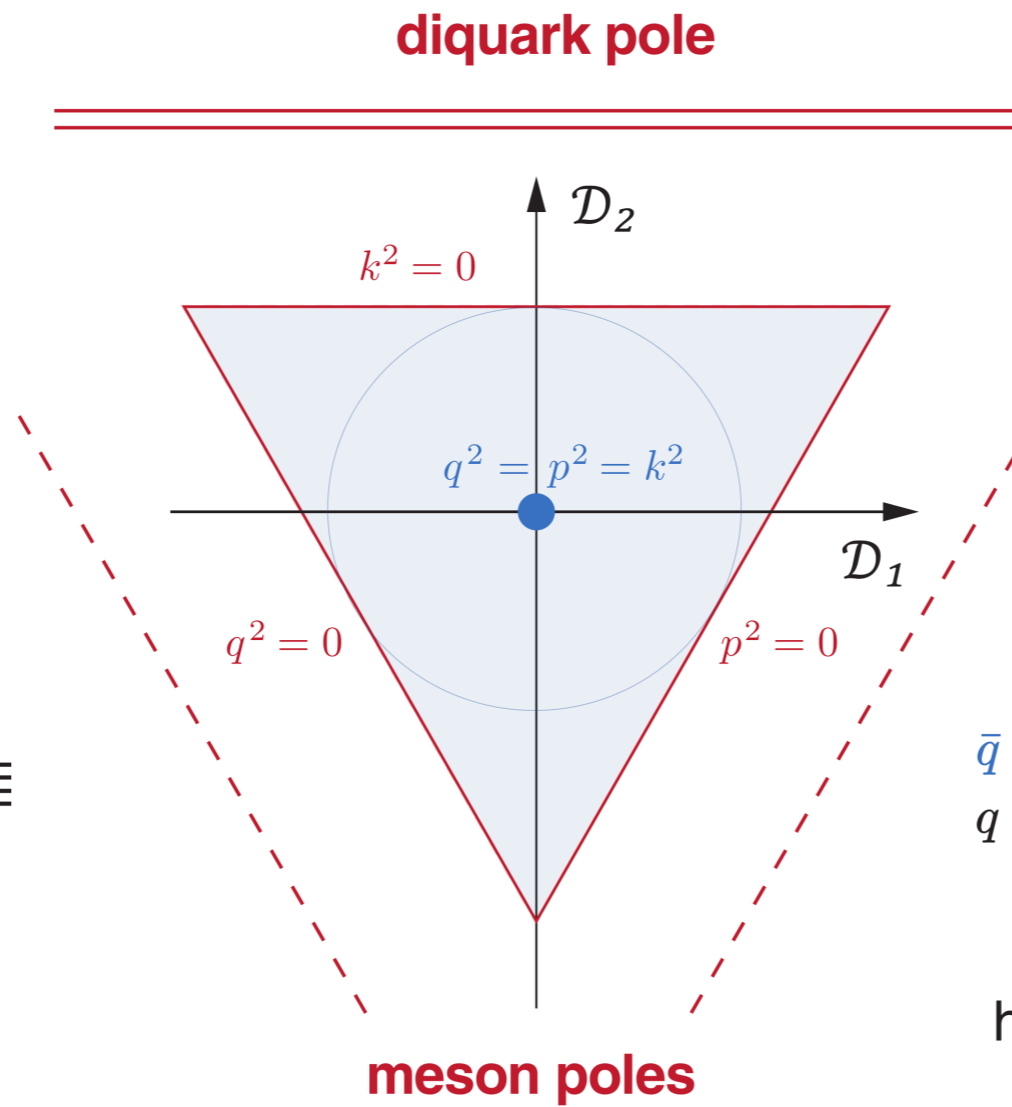
diquark-antidiquark



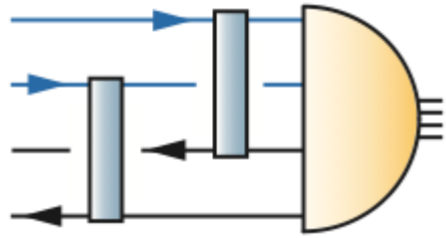
heavy-light mesons



hadrocharmonium



Bound state vs resonance: scalar four-quark states



$$\Gamma(S_0, \cancel{D_1}, \cancel{D_2})$$

without π -clustering

0

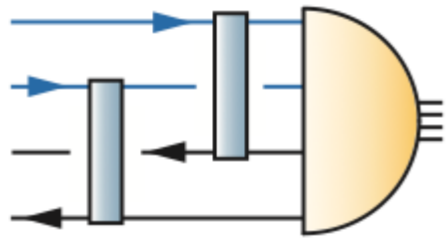
1200

$M_{\text{Tetra}} [MeV]$

Bound state of
four massive quarks

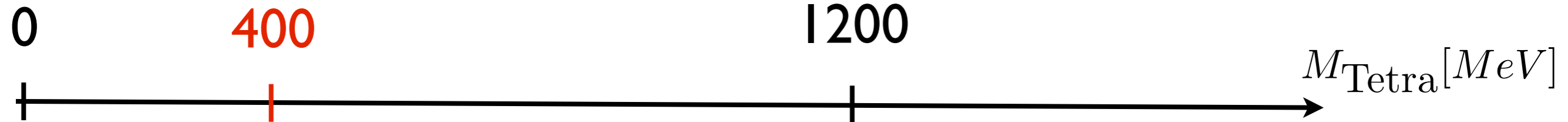
Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Bound state vs resonance: scalar four-quark states



$$\Gamma(S_0, \mathcal{D}_1, \mathcal{D}_2)$$

without π -clustering

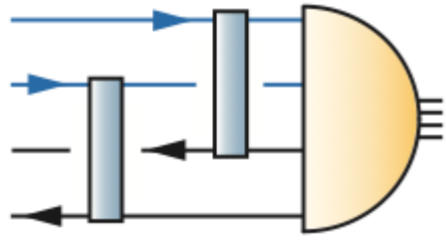


with π -clustering
Two-pion resonance

Bound state of
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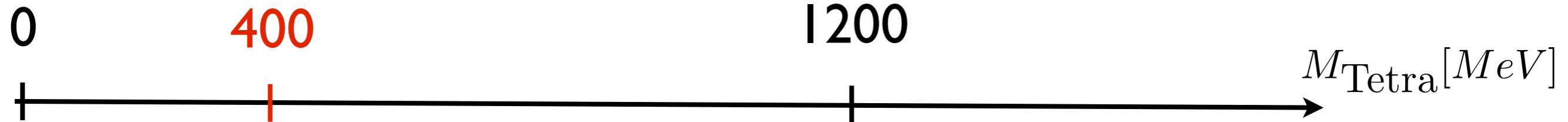
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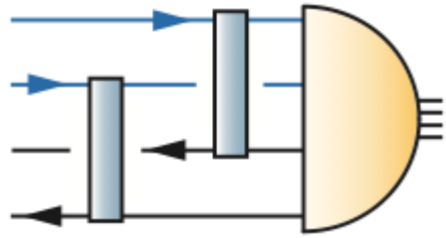
Two-pion resonance

Bound state of
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→ identify with $f_0(500)$ (' σ -meson')

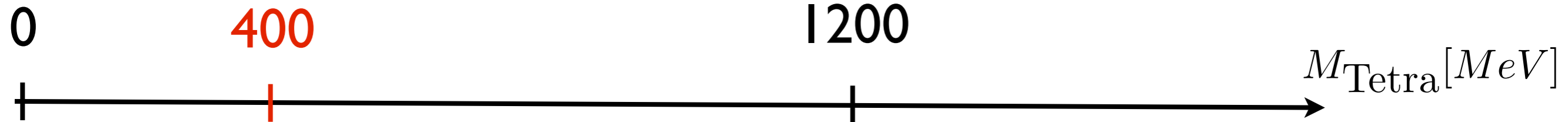
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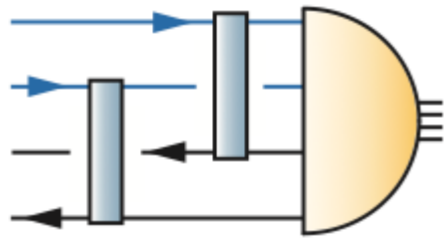
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with strange quarks: $m_\kappa \sim 750 \text{ MeV}$

$m_{a_0, f_0} \sim 1080 \text{ MeV}$

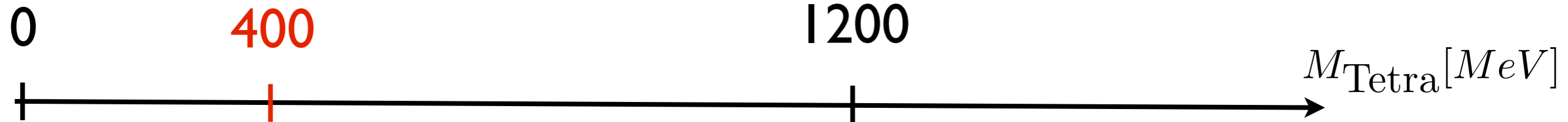
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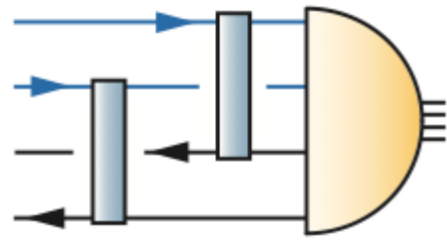
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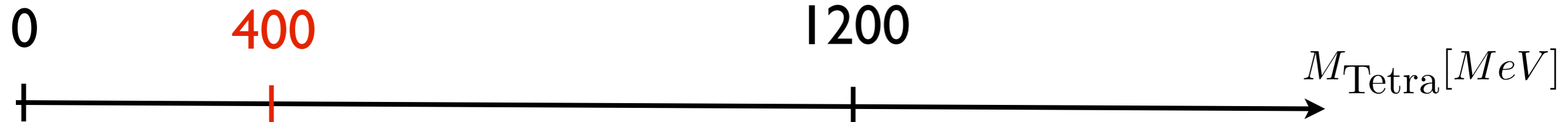
Meson-meson components dominate over diquarks !

Bound state vs resonance: scalar four-quark states



$$\Gamma(S_0, \mathcal{D}_1, \mathcal{D}_2)$$

without π -clustering



with π -clustering

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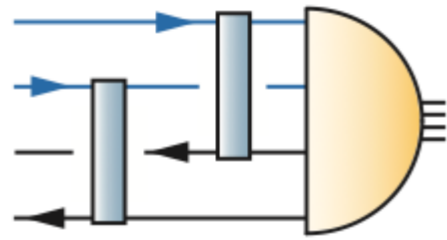
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Mixing with $q\bar{q}$: small effect

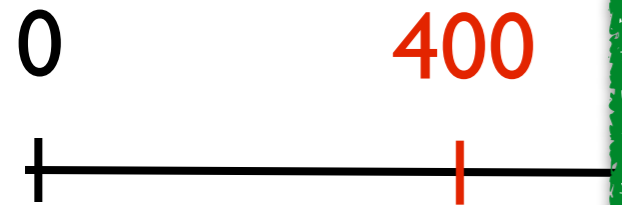
Santowsky, Eichmann, CF, Wallbott and Williams,
PRD 102 (2020) no.5, 056014, arXiv:2007.06495.

Bound state vs resonance: scalar four-quark states



$$\Gamma(S_0, \mathcal{D}_1, \mathcal{D}_2)$$

without π clustering



with π -clusters

Two-pion resonance

- model independent:
meson clusters more important than diquark
(color factor !)

→ identify with $f_0(500)$ (' σ -meson')

with strange quarks: $m_\kappa \sim 750 \text{ MeV}$

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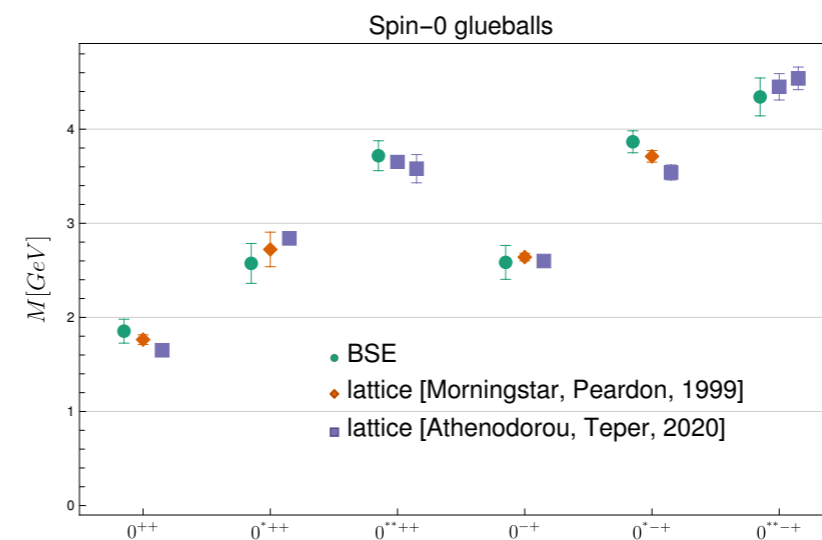
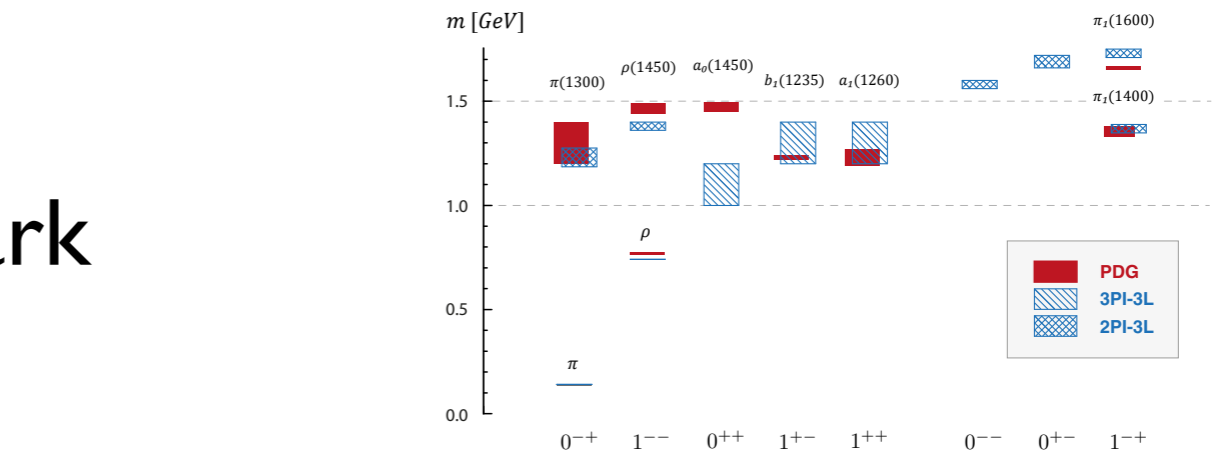
1. Light states: glueballs, two-quark and four-quark states

Williams, CF, Heupel, PRD 93 (2016) 034026 [arXiv:1512.00455]

Eichmann, CF, Heupel, PLB 753 (2016) 282 [arXiv:1508.07178]

Heupel, Eichmann, CF, PLB 718 (2021) 545 [arXiv:1206.5129]

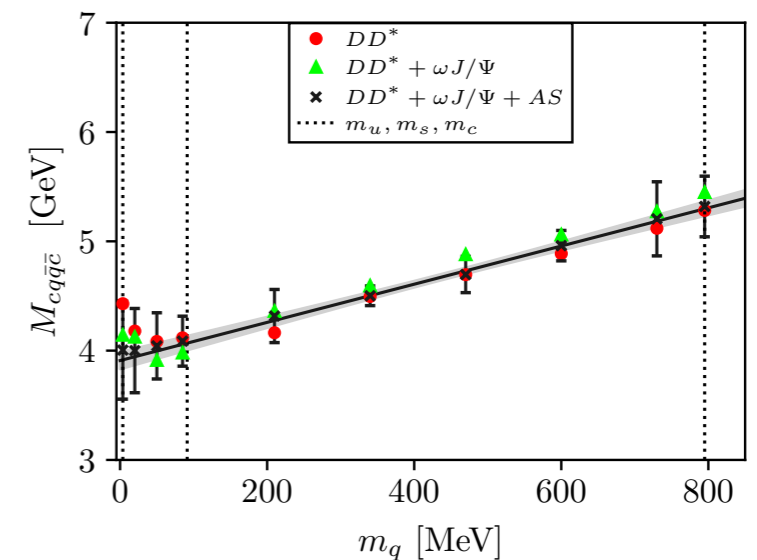
CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) 111077 [arXiv:2004.00415]

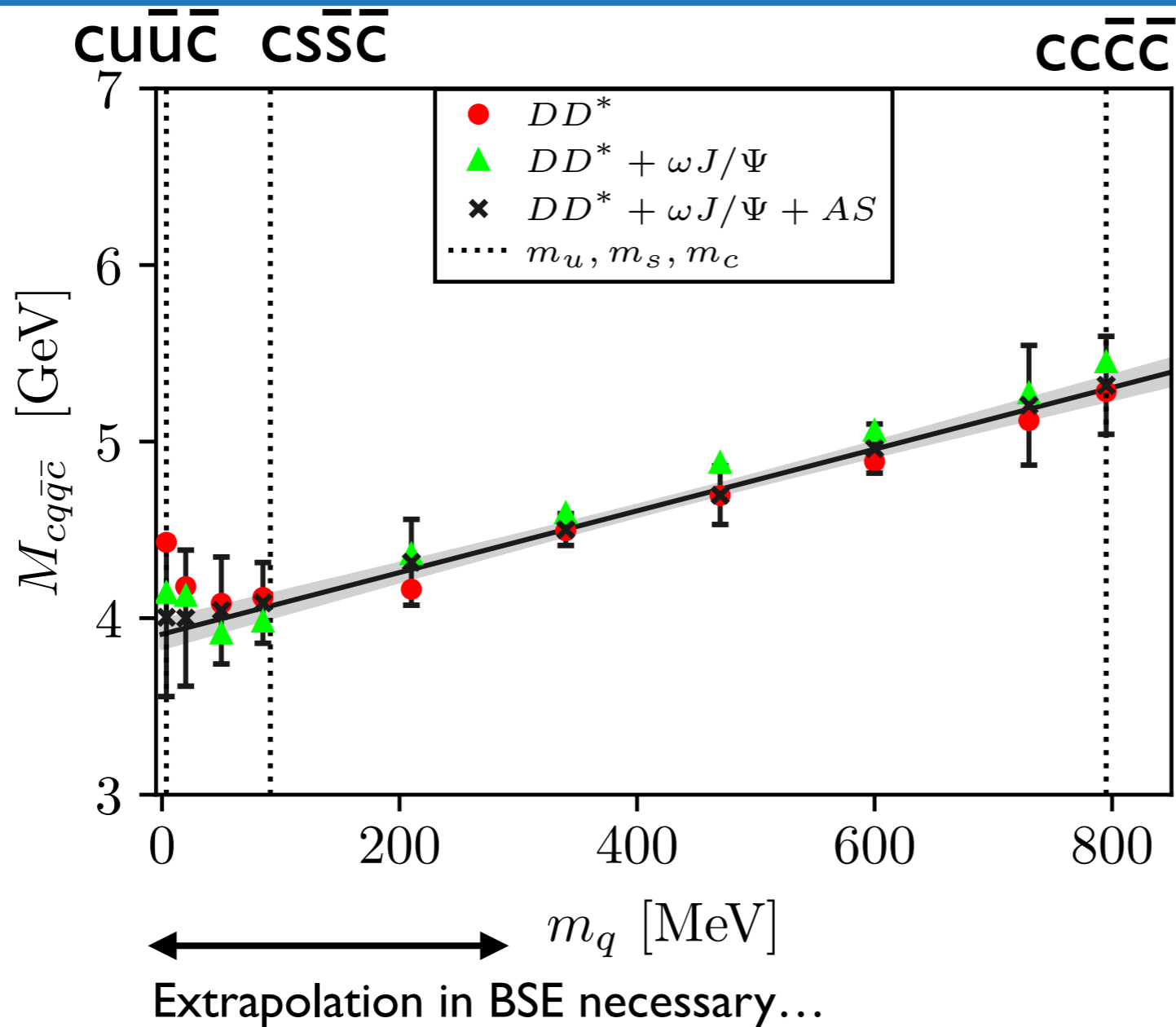
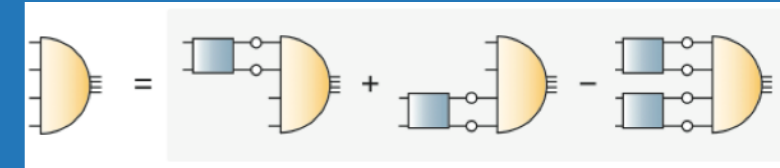


2. Heavy-light four-quark states: X(3872) and more...

Wallbott, Eichmann and CF, PRD100 (2019) no.1, 014033, [arXiv:1905.02615]

Wallbott, Eichmann and CF, PRD102 (2020) no.5, 051501, [arXiv:2003.12407]





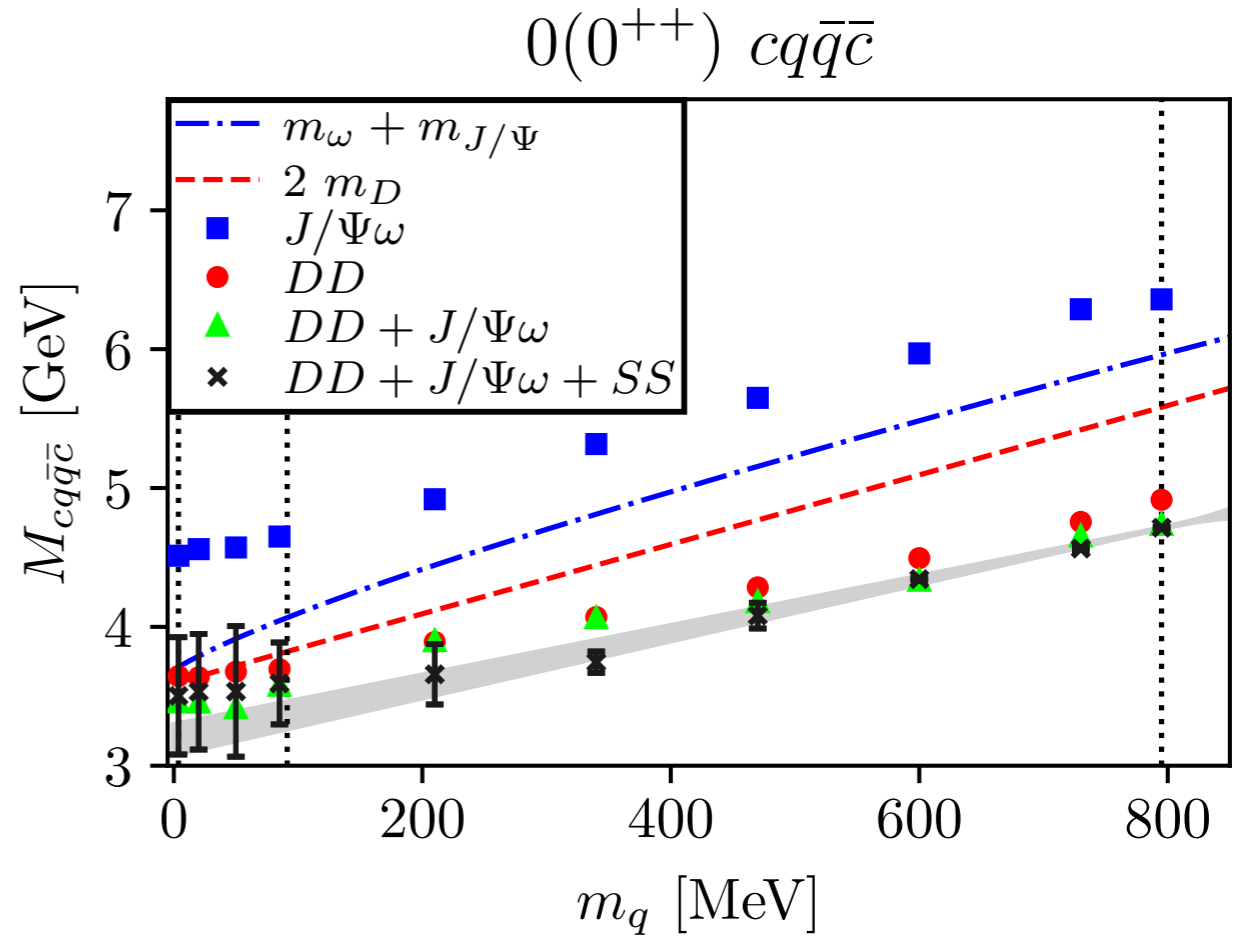
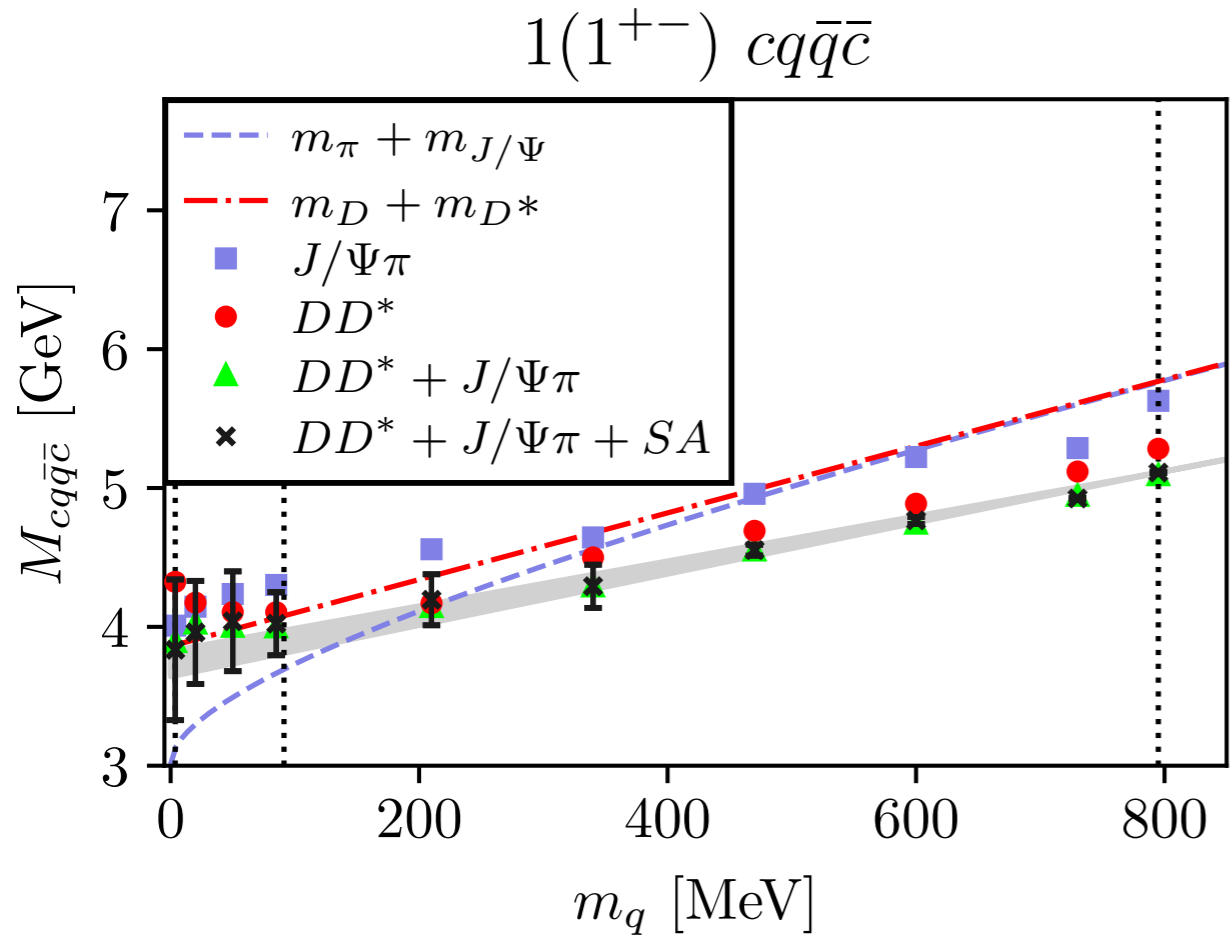
m_c fixed
 m_q varied

- DD^* components dominate !

$$M_{1^{++}}^{cq\bar{q}\bar{c}} = 3916(74) \text{ MeV} \longrightarrow X(3872)$$

Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]

$J^{PC} = 1^{+-}$ and 0^{++}

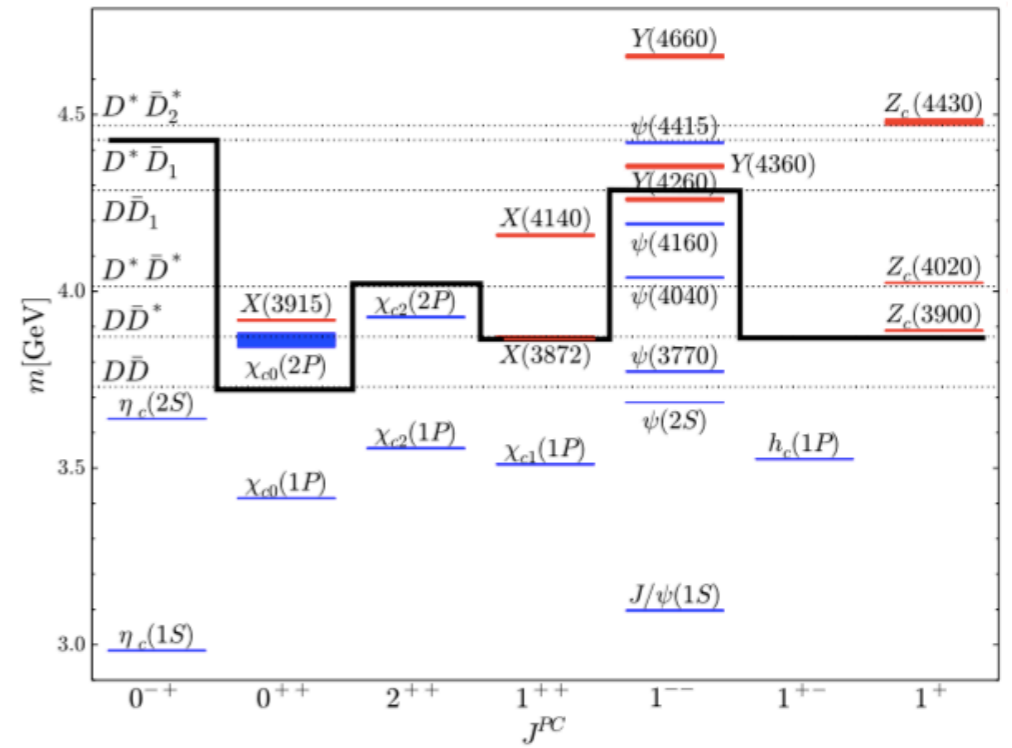


Wallbott, Eichmann and CF, PRD 102 (2020)no.5, 051501, arXiv:2003.12407

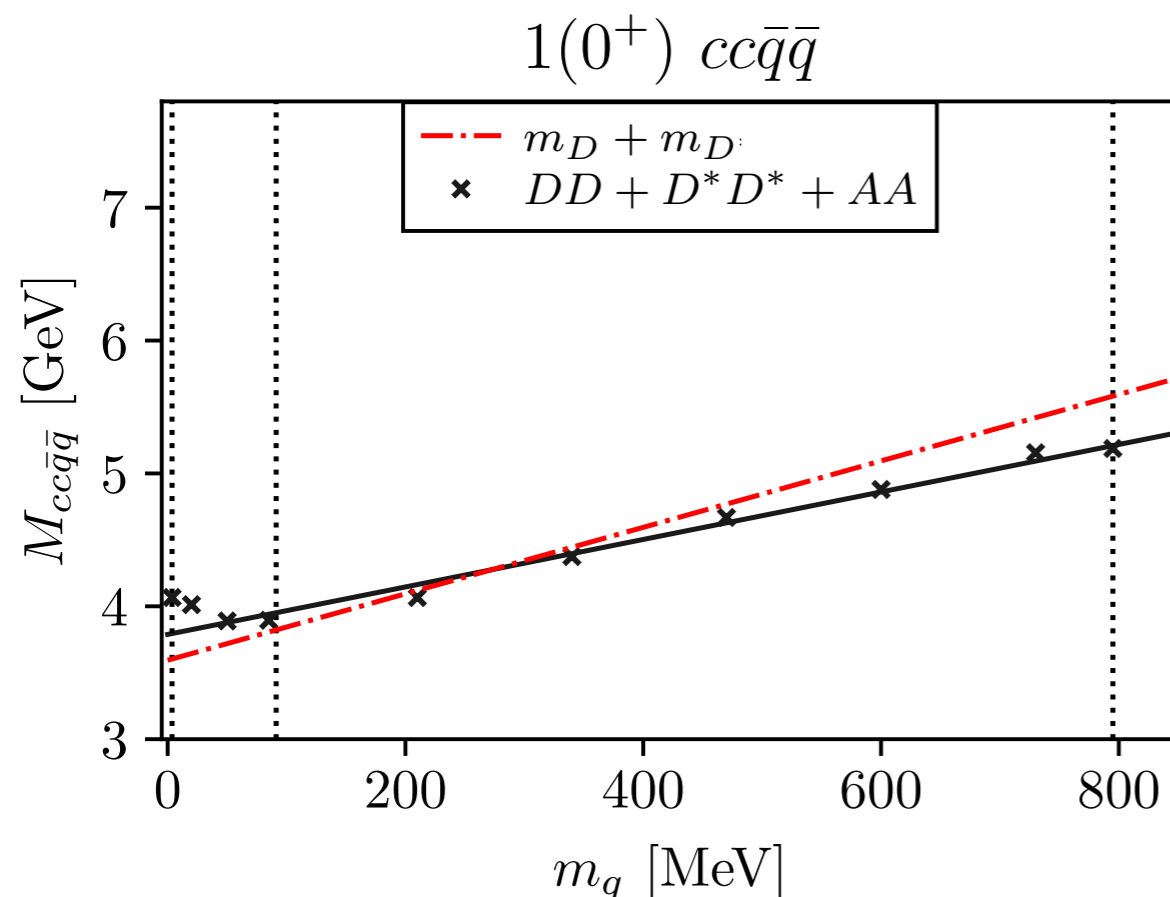
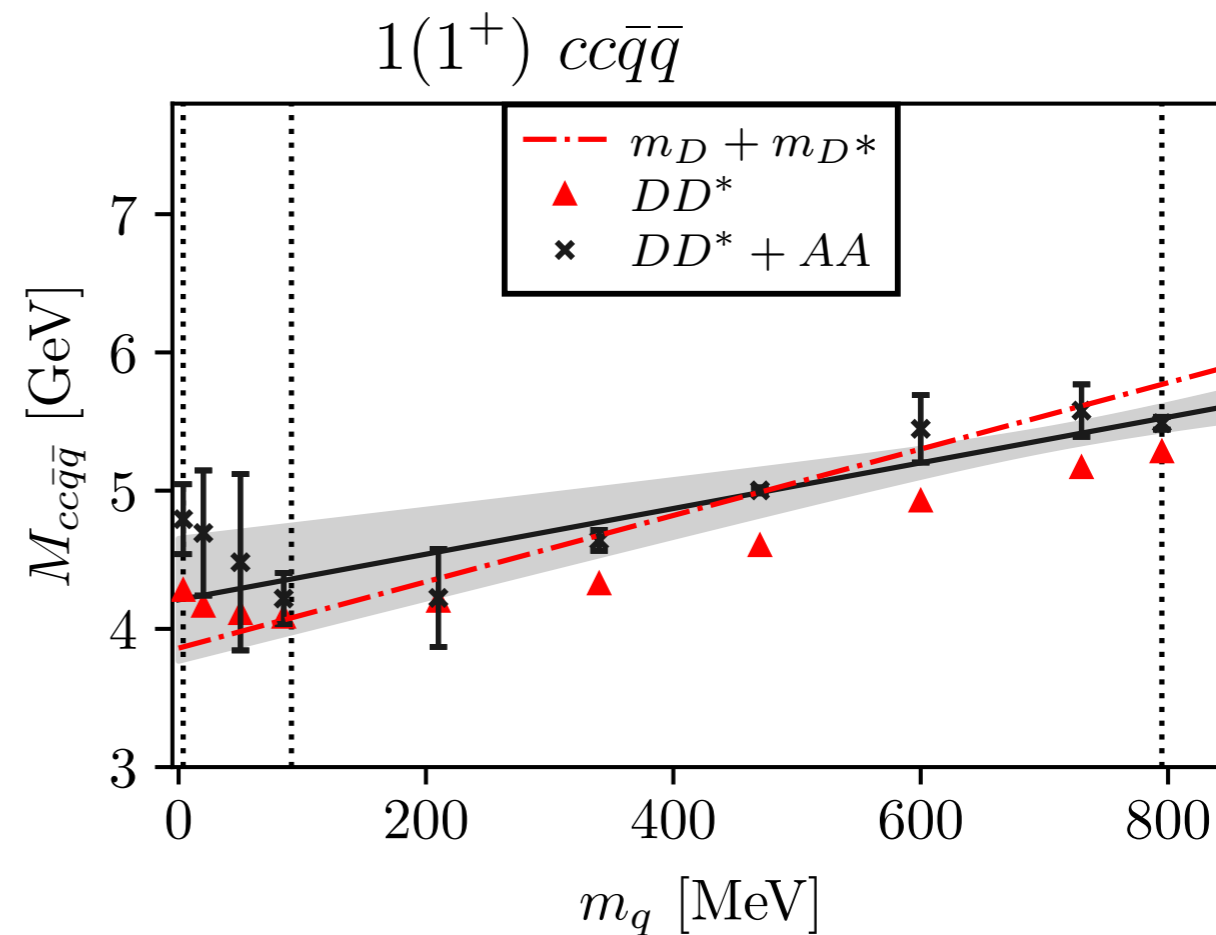
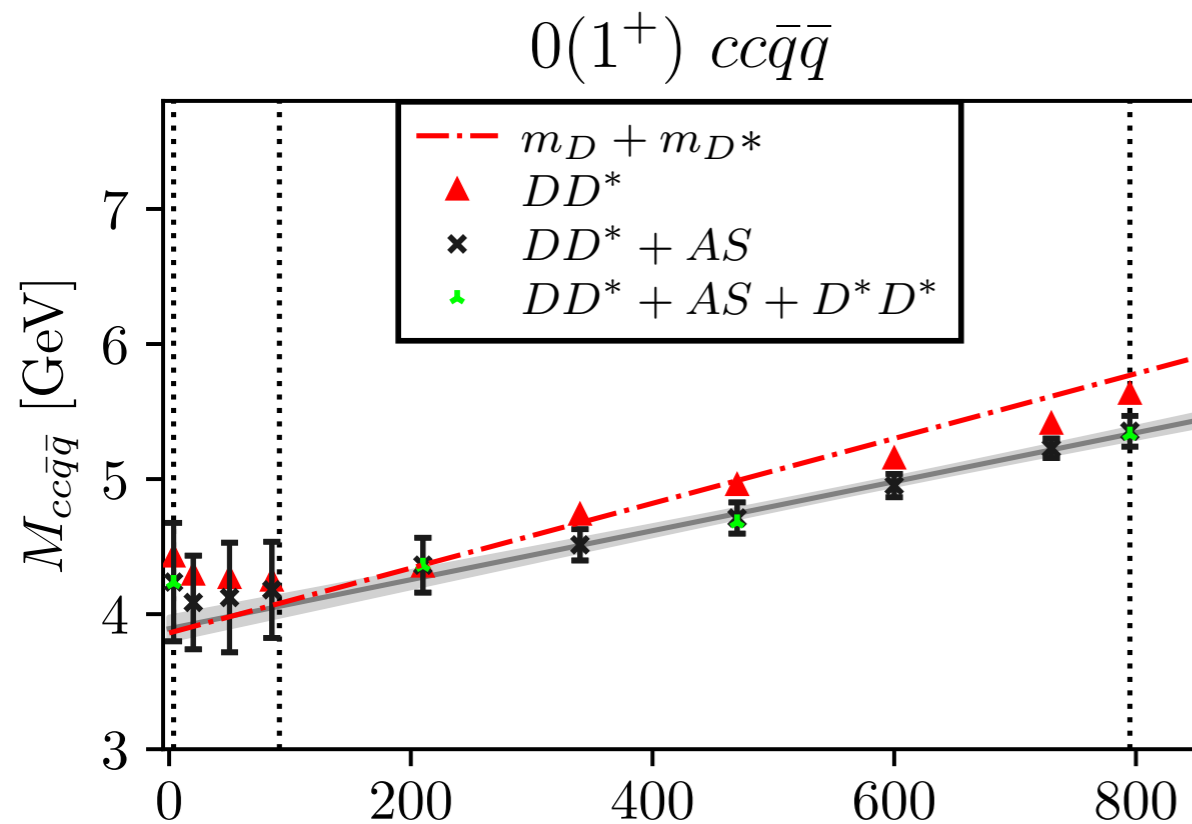
$$M_{1^{+-}}^{cq\bar{q}\bar{c}} = 3741(91) \rightarrow Z(3900)$$

$$M_{0^{++}}^{cq\bar{q}\bar{c}} = 3195(107) \rightarrow ?$$

mass pattern matches molecule picture of Cleven et al. PRD 92 (2015) 014005:



Open charm four-quark states



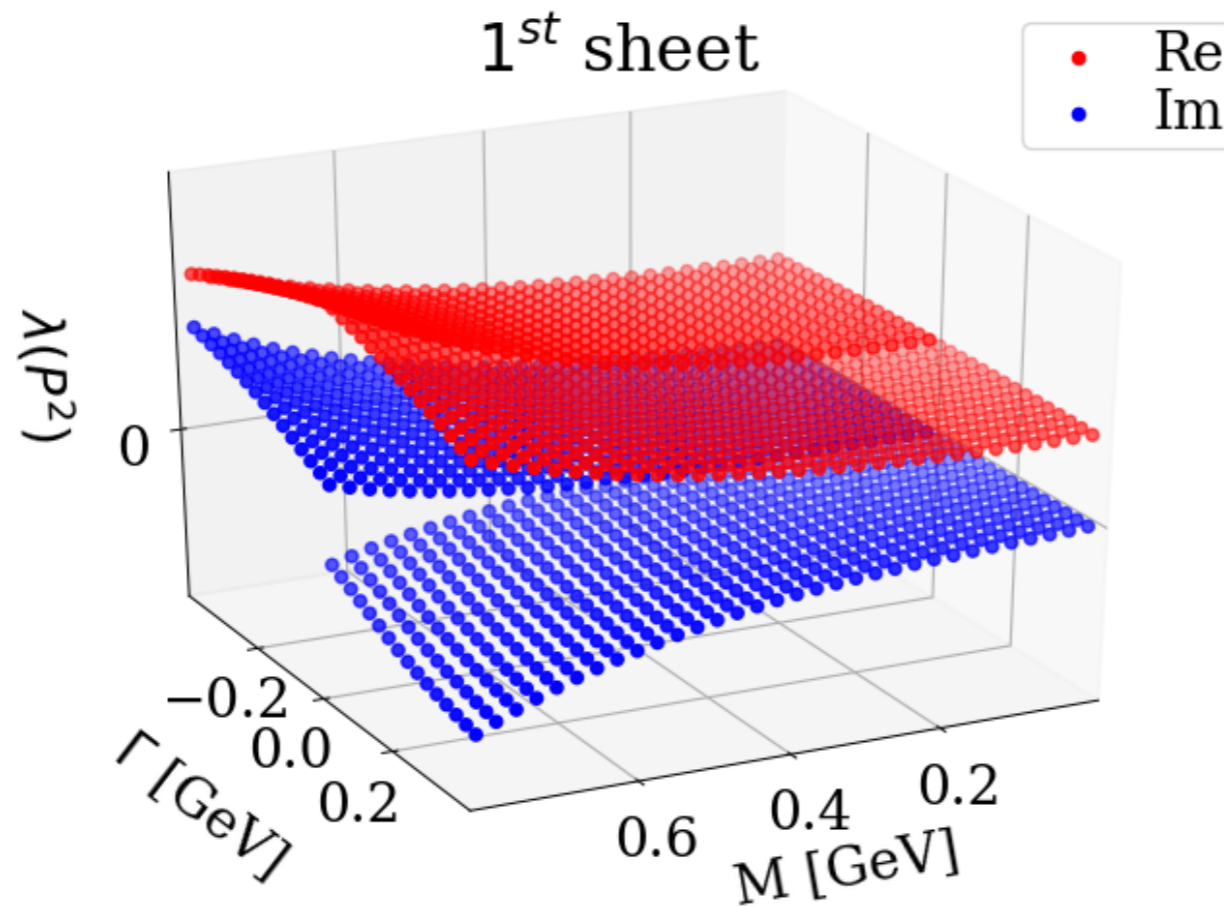
● **DD(*) and diquarks important!**

Wallbott, Eichmann and CF, PRD102 (2020)no.5, 051501, arXiv:2003.12407

Work to do...

- improve two-body interactions
- further study mixing with $q\bar{q}$ in $I=0$ sector
- solve four-body BSE in the complex momentum plane

Santowsky, Eichmann, CF, Wallbott and Williams,
PRD 102 (2020) no.5, 056014, arXiv:2007.06495.



successful for ρ -meson:

Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]

Internal dynamics very important !!

Glueballs:

- First quantitatively reliable results using very involved truncation

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]

Four-quark states:

- Dynamical description of σ : π - π resonance Eichmann, CF, Heupel, PLB 753 (2016) 282-287
- Dynamical description of $X(3872)$ and $Z(3900)$: DD^* dominated
- First results in open charm channels Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]
Wallbott, Eichmann and CF, PRD102 (2020) 051501, [2003.12407]
- Mixing with $q\bar{q}$ studied for light mesons Santowsky, Eichmann, CF, Wallbott and Williams,
PRD 102 (2020) no.5, 056014, [2007.06495].

Mini-Review: Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) 4 38, [2008.10240]

Rainbow-ladder model for quark-gluon interaction



Combine **gluon** with **quark-gluon vertex**:

$$\Gamma^\mu(p, k) = \sum_{i=1,12} \tau_i(p, k) T_i^\mu$$

$$\sim \gamma^\mu \tau(k^2)$$

“approximation” !

$$D^{\mu\nu}(k) = \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) \frac{Z(k^2)}{k^2}$$

$$\frac{g^2}{4\pi} \tau(k^2) Z(k^2) \sim \alpha(k^2)$$

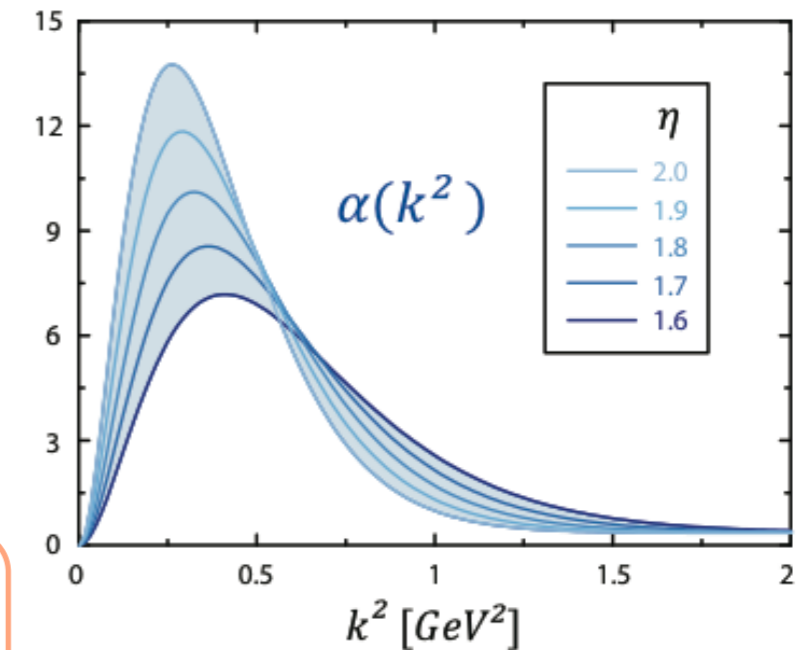
Rainbow-ladder model for quark-gluon interaction



Combine **gluon** with **quark-gluon vertex**:

effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2} \right) e^{-\eta^2 \left(\frac{k^2}{\Lambda^2} \right)} + \alpha_{UV}(k^2)$$

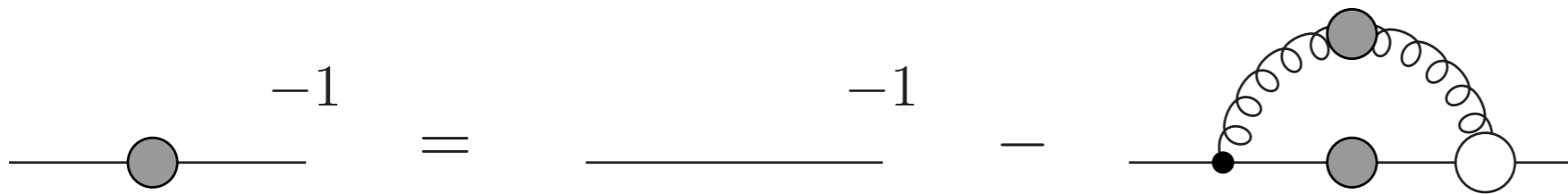


Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

- scale Λ from f_π , masses $m_u=m_d, m_s$ from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : results almost independent
- qualitatively similar to explicit calc.

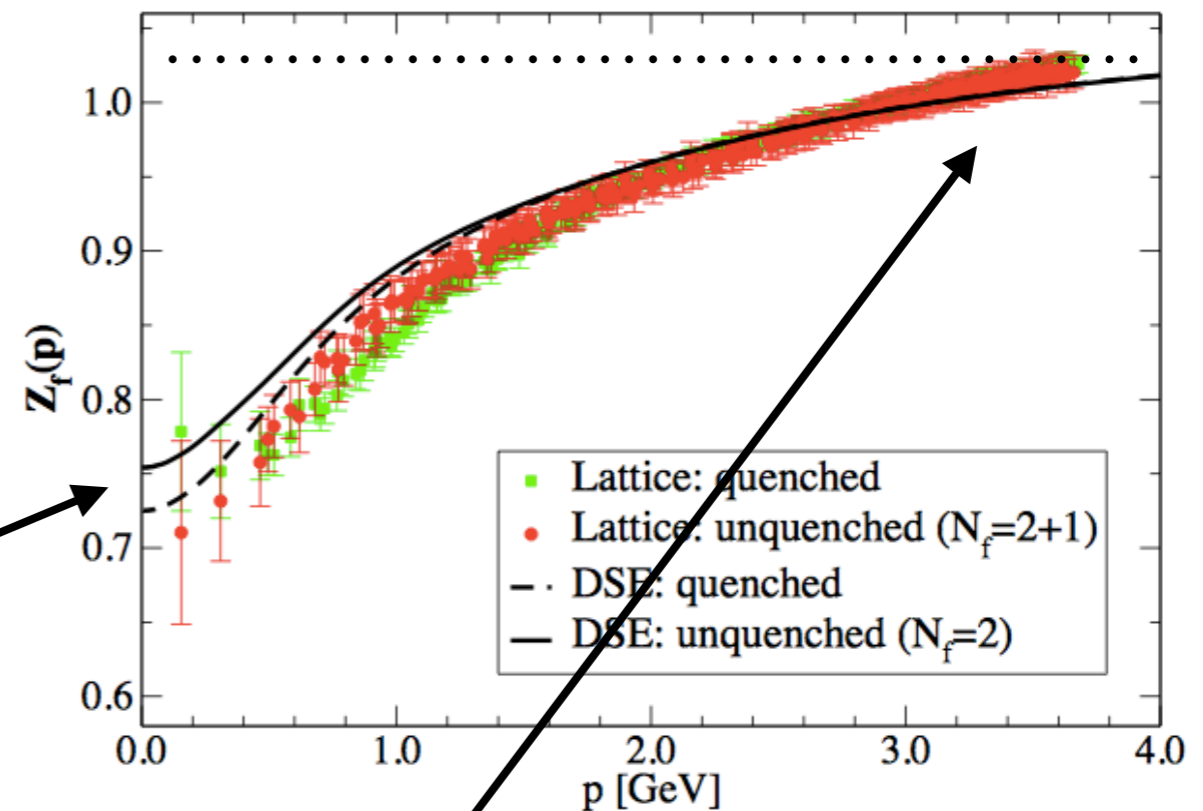
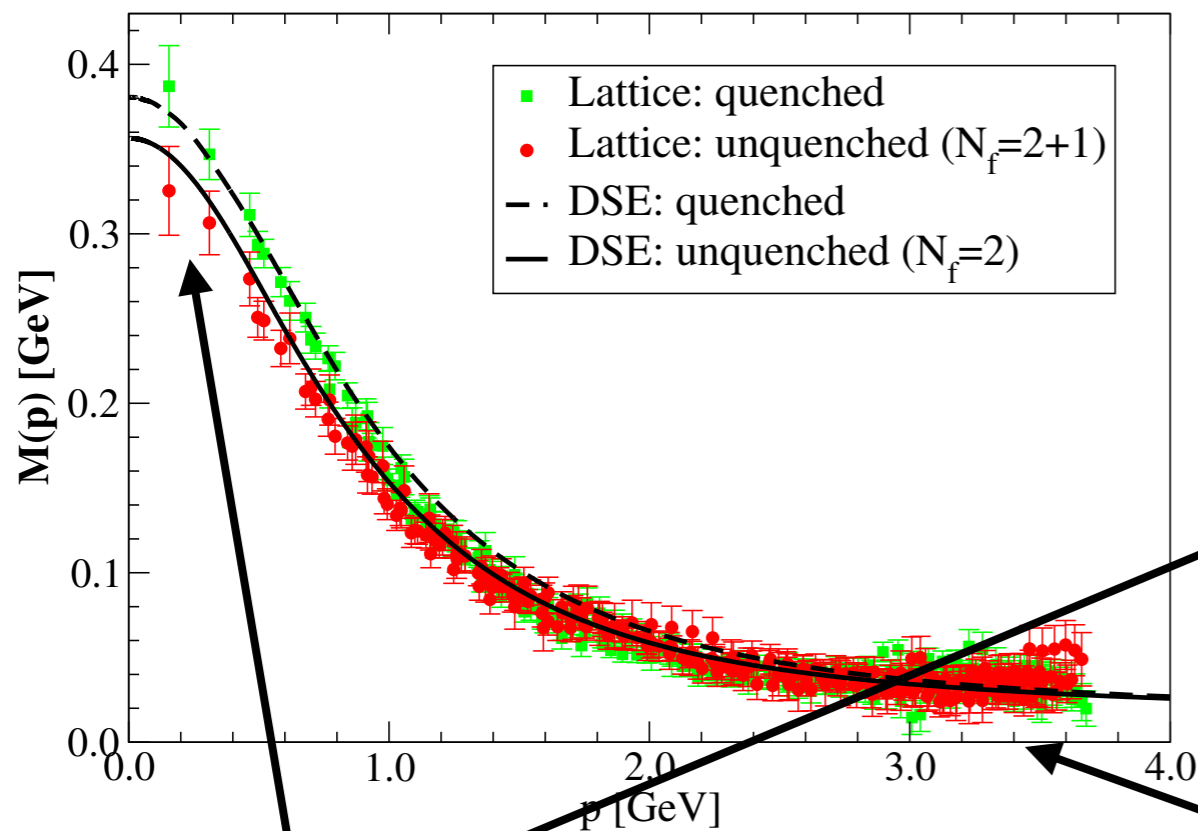
Williams, EPJA 51 (2015) 5, 57.
 Sanchis-Alepuz, Williams, PLB 749 (2015) 592;
 Mitter, Pawłowski and Strodthoff, PRD 91 (2015) 054035
 Williams, CF, Heupel, PRD93 (2016) 034026, and refs. therein

Quarks: mass from interaction



$$S(p) = Z_f(p^2) \frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

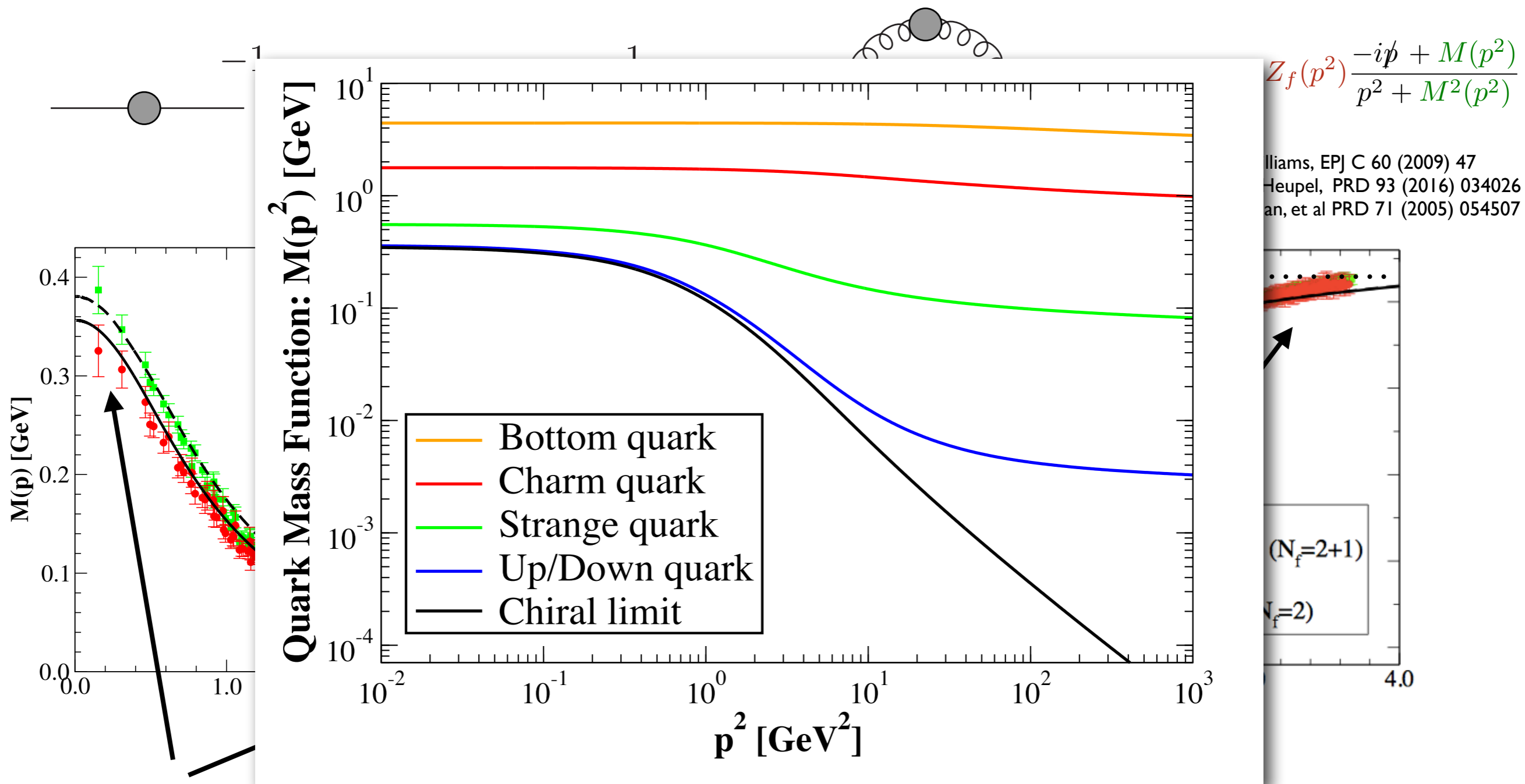
DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
 Williams, CF, Heupel, PRD 93 (2016) 034026
 Lattice: P. O. Bowman, et al PRD 71 (2005) 054507



‘constituent quark’:
 large mass; very composite

‘current quark’:
 - small mass; non-composite

Quarks: mass from interaction



‘constituent quark’:
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