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Study the nature of $f_0(980)$ and $a_0(980)$

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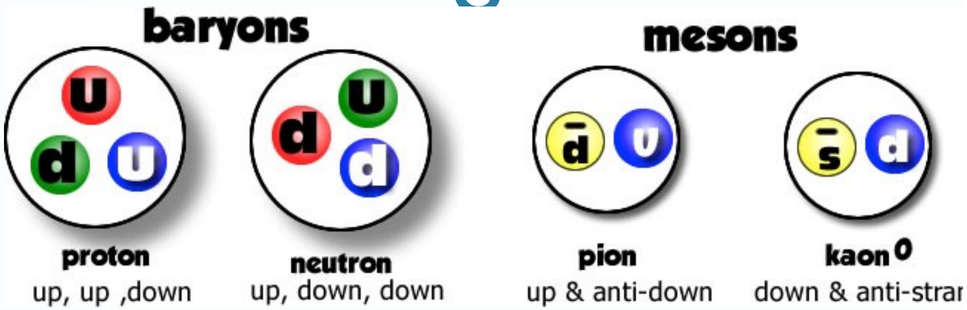
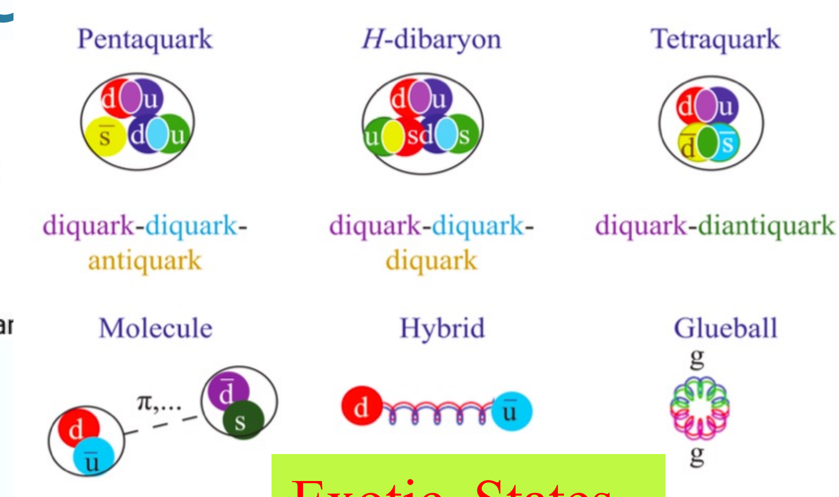
§1. Introduction



Quark model



Exotic States

H.-X. Chen, W. Chen, X. Liu and S.-L. Zhu, **Phys. Rept. 639** (2016) 1.
A. Esposito, A. Pilloni and A. D. Polosa, **Phys. Rept. 668** (2016) 1.
F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao and B.-S. Zou, **Rev. Mod. Phys. 90** (2018) 0115004.
S. L. Olsen, T. Skwarnicki and D. Zieminska, **Rev. Mod. Phys. 90** (2018) 0115003.
C. Z. Yuan, **Int. J. Mod. Phys. A 33**,1830018 (2018).
Y. R. Liu, H. X. Chen, W. Chen, X. Liu and S. L. Zhu, **Prog. Part. Nucl. Phys. 107**, 237 (2019) .
N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan, **Phys. Rept. 873** (2020) 1.

Long issue for their nature: $f_0(980)$ $a_0(980)$

- Normal qqbar state

D. Morgan, Phys. Lett. 51B, 71 (1974).
D. Morgan and M. R. Pennington, Z. Phys. C 48, 623 (1990).
N. A. Tornqvist and M. Roos, Phys. Rev. Lett. 76, 1575 (1996).

- Multiquark state

R. L. Jaffe, Phys. Rev. D 15, 267 (1977).
N. N. Achasov, Nucl. Phys. A 728, 425 (2003).
S.S.Agaev, K.Aziziand, H.Sundu,Phys.Lett.B781,279(2018).

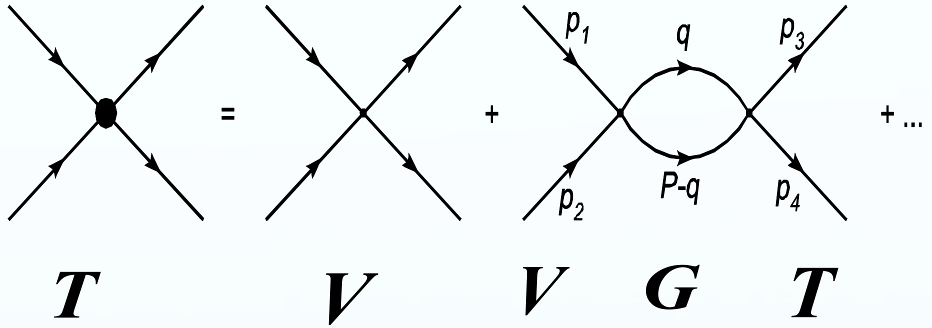
- KKbar molecules

J. D. Weinstein and N. Isgur, Phys. Rev. Lett. 48, 659 (1982).
G. Janssen, B. C. Pearce, K. Holinde and J. Speth, Phys. Rev. D 52, 2690 (1995) .
J. A. Oller and E. Oset, Nucl. Phys. A 620, 438 (1997).

§2. Formalism

- Chiral Unitary Approach (ChUA)： coupled channel approach, solving Bethe-Salpeter (BS) equations.

$$T = V + V\,G\,T,\; T = [1 - V\,G]^{-1}\,V$$



where V matrix (potentials) can be evaluated from chiral Lagrangians.

J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438
E. Oset and A. Ramos, Nucl. Phys. A 635 (1998) 99
J. A. Oller and U. G. Meißner, Phys. Lett. B 500 (2001) 263

G is a diagonal matrix with the loop functions of each channels:

$$G_{ll}(s) = i\int \frac{d^4q}{(2\pi)^4} \frac{1}{(P-q)^2-m_{l1}^2+i\varepsilon} \frac{1}{q^2-m_{l2}^2+i\varepsilon}$$

The coupled channel scattering amplitudes **T matrix satisfy the unitary** :

$$\mathrm{Im}\, T_{ij} = T_{in}\,\sigma_{nn}\,T_{nj}^*$$

$$\sigma_{nn} \equiv \mathrm{Im}\, G_{nn} = -\frac{q_{cm}}{8\pi\sqrt{s}}\theta(s-(m_1+m_2)^2))$$

To search the poles of the resonances, we should extrapolate the scattering amplitudes to the second Riemann sheets:

$$G_{ll}^{II}(s) = G_{ll}^{I}(s) + i\,\frac{q_{cm}}{4\pi\sqrt{s}}$$

To understand more properties of the resonances, we first evaluate the sum rule for the composite state

$$-\sum_i g_i^2 \left[\frac{dG_i}{ds}\right]_{s=s_{pole}} = 1 - Z$$

The wave function and the form factor are given by

$$\phi(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \frac{4\pi}{r} \frac{1}{C} \int_{q_{\max}} pdp \sin(pr) \times \frac{\Theta\left(q_{\max} - |\vec{p}|\right)}{E - \omega_1(\vec{p}) - \omega_2(\vec{p})} \frac{m_V^2}{\vec{p}^2 + m_V^2}$$

$$F(\vec{q}) = \int d^3\vec{r}\phi(\vec{r})\phi^*(\vec{r})e^{-i\vec{q}'\cdot\vec{r}}$$
$$= \int d^3\vec{p} \frac{\theta(\Lambda - p)\,\theta(\Lambda - |\vec{p} - \vec{q}|)}{[E - \omega_1(p) - \omega_2(p)]\,[E - \omega_1(\vec{p} - \vec{q}) - \omega_2(\vec{p} - \vec{q})]}$$

With the form factor obtained, the radius can be evaluated by

$$\langle r^2 \rangle = -6 \left[\frac{\mathrm{d}F(q)}{\mathrm{d}q^2}\right]_{q^2=0}$$

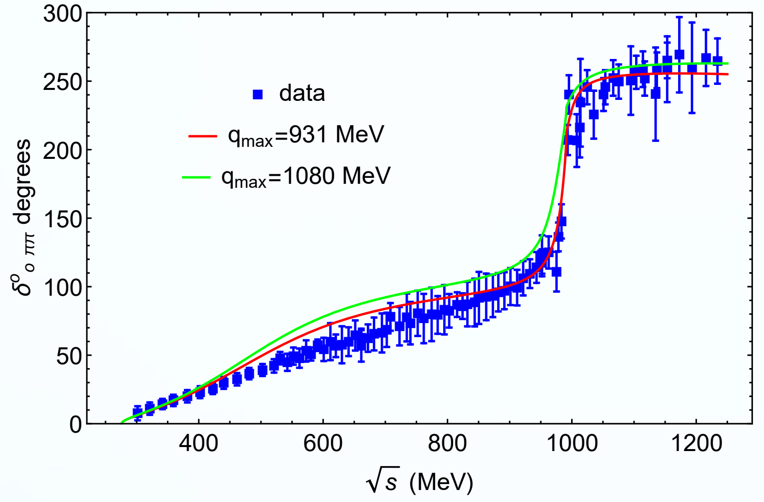
Or one can use the one from the tail of the wave functions

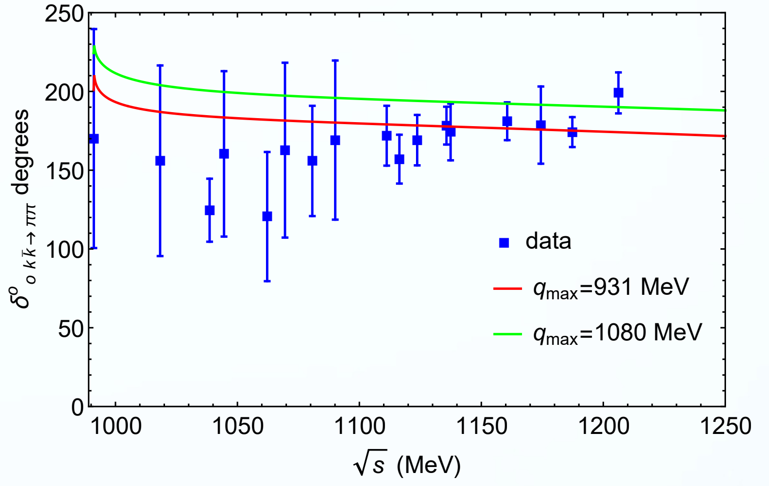
$$\langle r^2 \rangle_i = \frac{-g_i^2 \left[\frac{\mathrm{d}G_i(s)}{\mathrm{d}s}\right]_{s=s_{pole}}}{4\mu_i B_{\mathrm{E},i}}$$

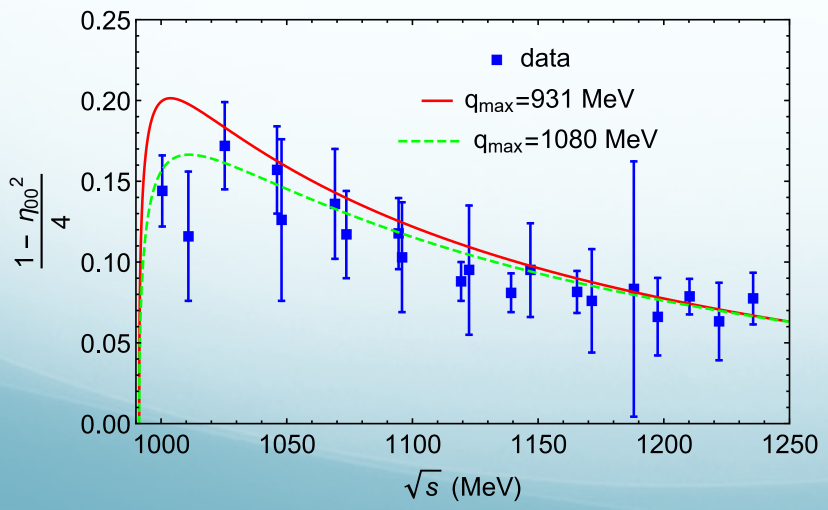
T. Sekihara and T. Hyodo, Phys. Rev. C 87, 045202 (2013).

§3. Results

1) The results of the coupled channel interaction







The central value of the cutoff

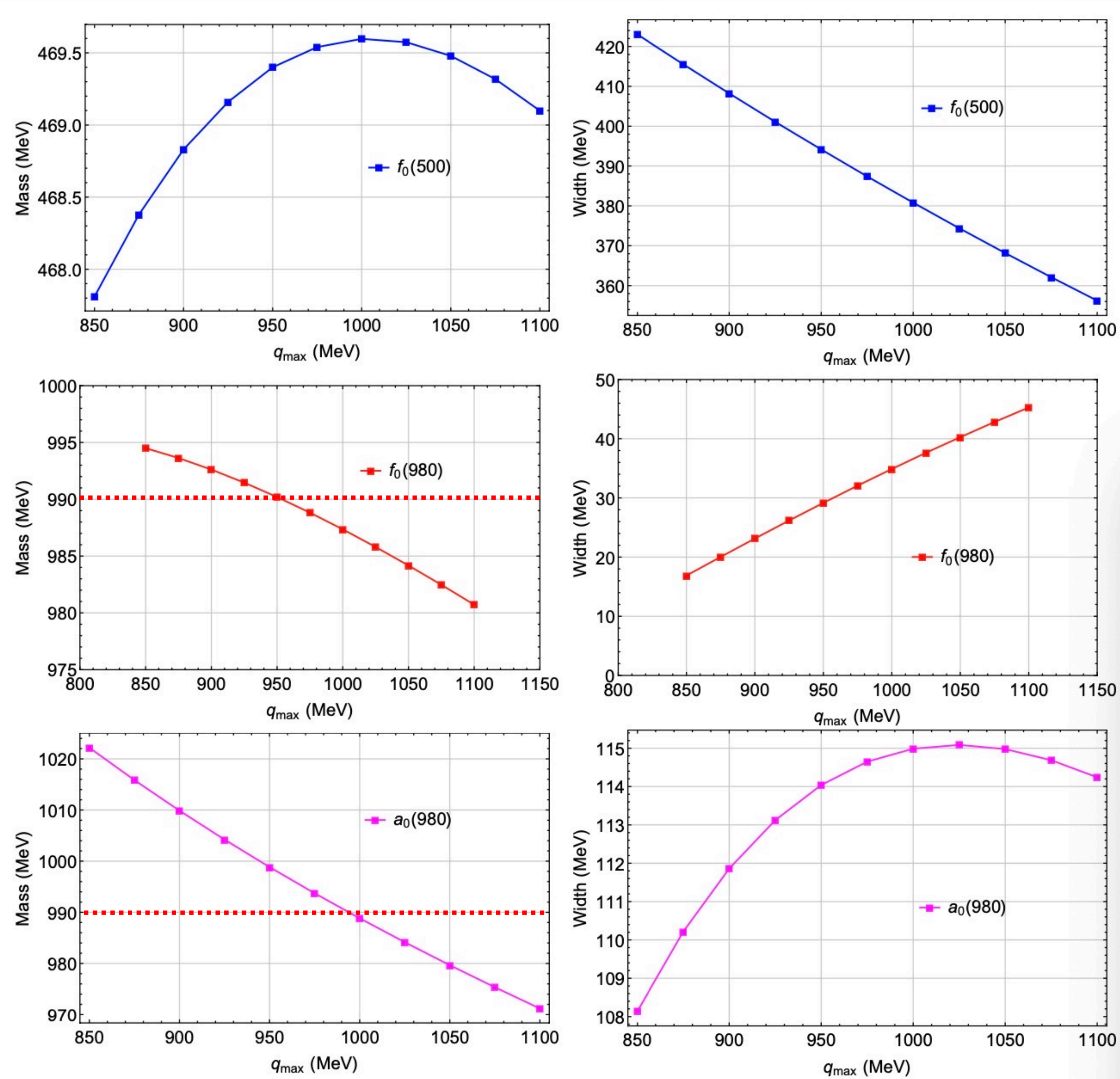
$$q_{max} = 931 \text{ MeV}$$

which is taken from

CWX, U.-G. Meißner and J. A. Oller, Eur. Phys. J. A 56, 23 (2020).

Pole trajectories for varying the cutoff

threshold



Couplings and the conpositeness $I = 0$ sector

$q_{max} = 931$ MeV	$g_{K\bar{K}} g_{K\bar{K}}$ (GeV^2)	$\lvert g_{K\bar{K}} \rvert$ (GeV)	$g_{\pi\pi} g_{\pi\pi}$ (GeV^2)	$\lvert g_{\pi\pi} \rvert$ (GeV)
σ : $469.23 + 199.70i$	$-1.05 + 1.72i$	1.42	$-3.49 + 8.20i$	2.98
f_0 : $991.17 + 13.45i$	$10.92 - 10.91i$	3.92	$-1.76 + 0.70i$	1.37
$q_{max} = 1080$ MeV				
σ : $469.28 + 180.46i$	$-0.80 + 1.86i$	1.42	$-2.0 + 8.28i$	2.92
f_0 : $982.13 + 21.67i$	$16.15 - 10.55i$	4.39	$-2.34 + 1.11i$	1.60

$q_{max} = 931$ MeV	$(1 - Z)_{K\bar{K}}$	$\lvert (1 - Z)_{K\bar{K}} \rvert$	$(1 - Z)_{\pi\pi}$	$\lvert (1 - Z)_{\pi\pi} \rvert$
σ : $469.23 + 199.70i$	$-0.01 + 0.01i$	0.01	$-0.13 - 0.37i$	0.40
f_0 : $991.17 + 13.45i$	$0.79 + 0.12i$	0.80	$0.02 - 0.01i$	0.02
$q_{max} = 1080$ MeV				
σ : $469.28 + 180.46i$	$-0.00 + 0.01i$	0.01	$-0.16 - 0.36i$	0.39
f_0 : $982.13 + 21.67i$	$0.70 + 0.11i$	0.70	$0.02 - 0.01i$	0.02

$I = 1$ sector

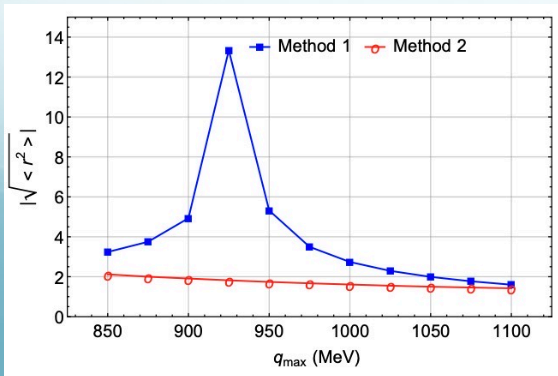
$q_{max} = 931$ MeV	$g_{K\bar{K}} g_{K\bar{K}}(\text{GeV}^2)$	$\lvert g_{K\bar{K}}\rvert(\text{GeV})$	$g_{\pi\eta} g_{\pi\eta}(\text{GeV}^2)$	$\lvert g_{\pi\eta}\rvert(\text{GeV})$
$a_0 : 1002.90 + 56.68i$	$24.17 - 9.22i$	5.08	$10.30 + 5.71i$	3.43
$q_{max} = 1080$ MeV				
$a_0 : 974.50 + 57.31i$	$21.83 - 3.28i$	4.78	$8.16 + 5.20i$	3.11

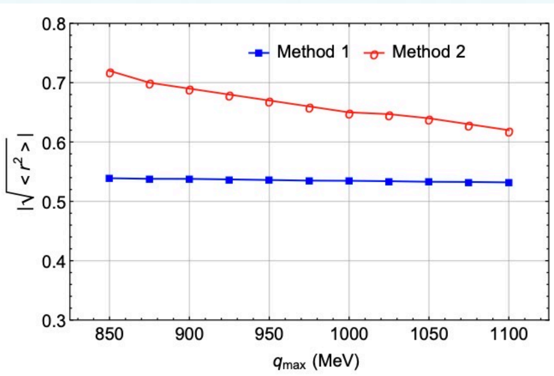
$q_{max} = 931$ MeV	$(1 - Z)_{K\bar{K}}$	$\lvert(1 - Z)_{K\bar{K}}\rvert$	$(1 - Z)_{\pi\eta}$	$\lvert(1 - Z)_{\pi\eta}\rvert$
$a_0 : 1002.90 + 56.68i$	$0.37 + 0.41i$	0.55	$-0.09 - 0.13i$	0.16
$q_{max} = 1080$ MeV				
$a_0 : 974.50 + 57.31i$	$0.34 + 0.29i$	0.45	$-0.07 - 0.12i$	0.14

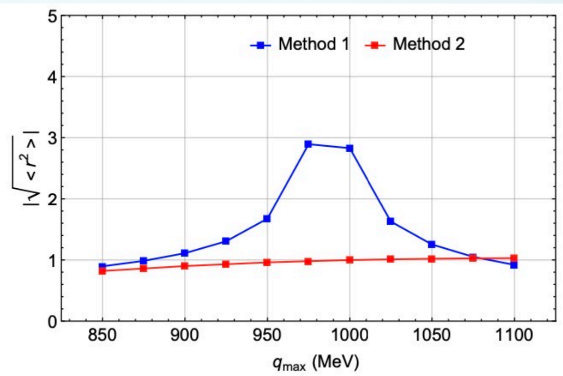
The radii of states

Resonances	$q_{max} = 931$ MeV	$\lvert\sqrt{\langle r^2\rangle}\rvert$	$q_{max} = 1080$ MeV	$\lvert\sqrt{\langle r^2\rangle}\rvert$
f_0	$1.42 + 1.10i$ fm	1.80 fm	$1.31 + 0.62i$ fm	1.45 fm
σ	$0.68 + 0.005i$ fm	0.68 fm	$0.63 + 0.04i$ fm	0.63 fm
a_0	$0.83 + 0.44i$ fm	0.94 fm	$0.96 + 0.35i$ fm	1.03 fm

Resonances	$q_{max} = 931$ MeV	$\lvert\sqrt{\langle r^2\rangle}\rvert$	$q_{max} = 1080$ MeV	$\lvert\sqrt{\langle r^2\rangle}\rvert$
f_0	$16.32 + 1.20i$ fm	16.36 fm	$1.73 + 0.13i$ fm	1.73 fm
σ	$0.43 + 0.31i$ fm	0.54 fm	$0.44 + 0.29i$ fm	0.53 fm
a_0	$0.56 - 1.25i$ fm	1.37 fm	$0.96 + 0.36i$ fm	1.02 fm

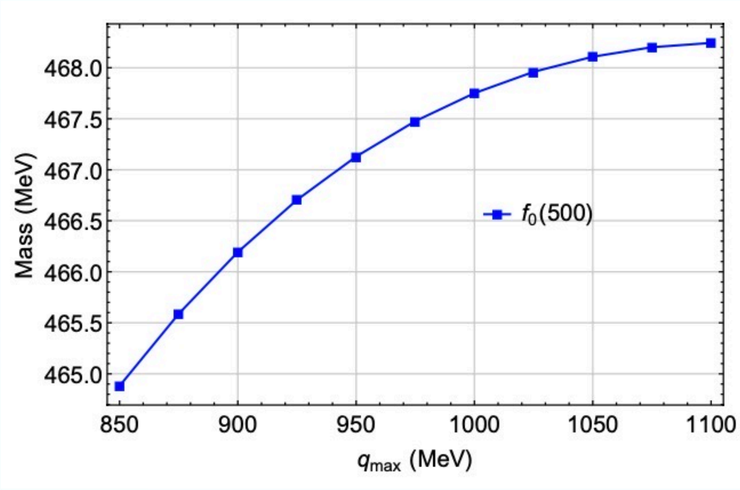


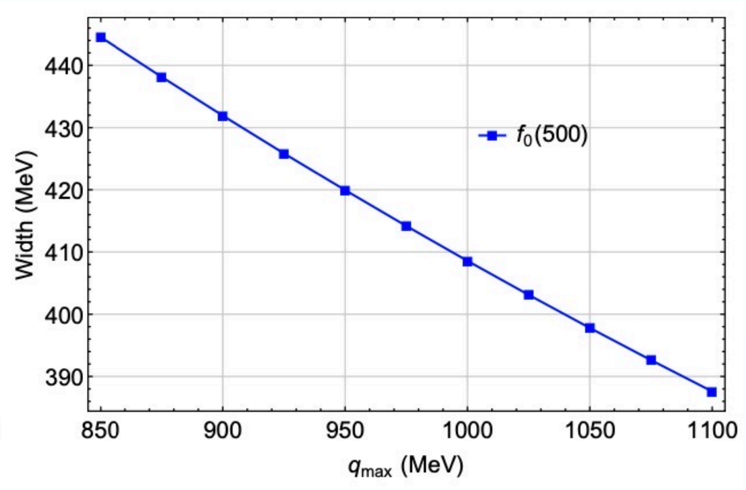


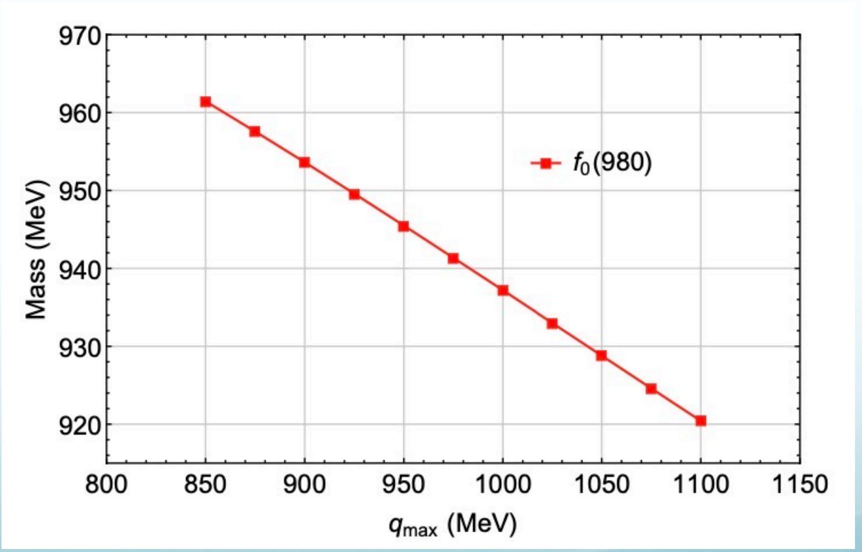


2) The results of the single channel interaction

Pole trajectories for varying the cutoff







The potential of KKbar is too weak to create a pole in I=1.

Couplings and the conpositeness

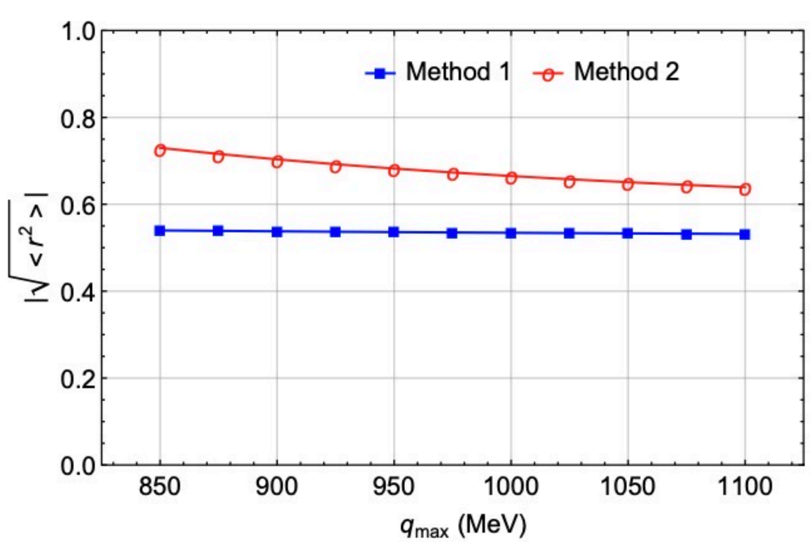
$q_{max} = 931$ MeV	$g_{K\bar{K}} g_{K\bar{K}}$(GeV^2)	$\lvert g_{K\bar{K}} \rvert$(GeV)	$g_{\pi\pi} g_{\pi\pi}$(GeV^2)	$\lvert g_{\pi\pi} \rvert$(GeV)
σ : $466.81 + 212.21i$	0	0	$-4.41 + 7.77i$	2.98
$f_0(980)$: 948.62	26.4	5.13	0	0
$q_{max} = 1080$ MeV				
σ : $468.213 + 195.8i$	0	0	$-3.20 + 8.05i$	2.942
$f_0(980)$: 923.77	29.8	5.45	0	0

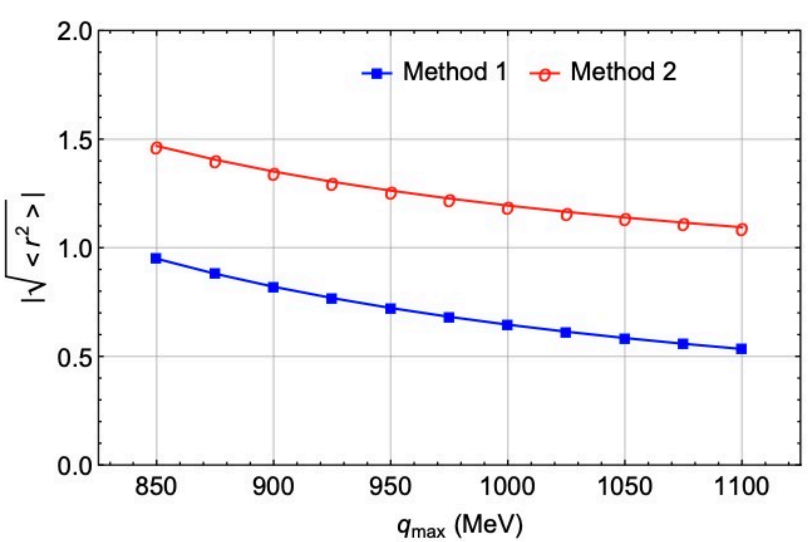
$q_{max} = 931$ MeV	$(1-Z)_{K\bar{K}}$	$\lvert(1-Z)_{K\bar{K}}\rvert$	$(1-Z)_{\pi\pi}$	$\lvert(1-Z)_{\pi\pi}\rvert$
σ : $467.13 + 209.968i$	0	0	$-0.11 - 0.37i$	0.39
$f_0(980)$: 948.62	0.62	0.62	0	0
$q_{max} = 1080$ MeV				
σ : $468.213 + 195.8i$	0	0	$-0.13 - 0.36i$	0.386
$f_0(980)$: 923.77	0.52	0.52	0	0

The radii of states

Resonances	$q_{max} = 931$ MeV	$\lvert\sqrt{\langle r^2\rangle}\rvert$	$q_{max} = 1080$ MeV	$\lvert\sqrt{\langle r^2\rangle}\rvert$
σ	$0.69 + 0.007\,i$ fm	0.69 fm	$0.64 + 0.03\,i$ fm	0.64 fm
$f_0(980)$	1.29 fm	1.29 fm	1.11 fm	1.11 fm

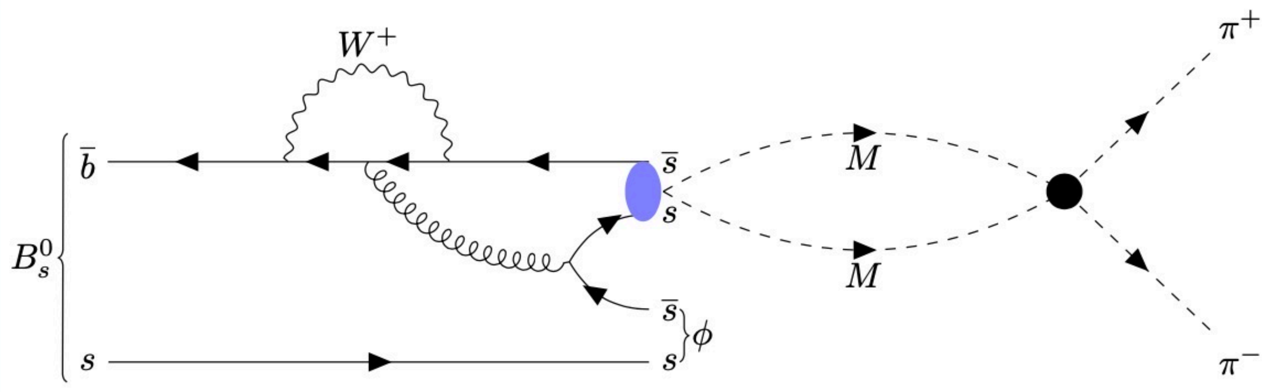
Resonances	$q_{max} = 931$ MeV	$\lvert\sqrt{\langle r^2\rangle}\rvert$	$q_{max} = 1080$ MeV	$\lvert\sqrt{\langle r^2\rangle}\rvert$
σ	$0.43 + 0.32\,i$ fm	0.54 fm	$0.43 + 0.30\,i$ fm	0.53 fm
$f_0(980)$	0.75 fm	0.75 fm	0.55 fm	0.55 fm

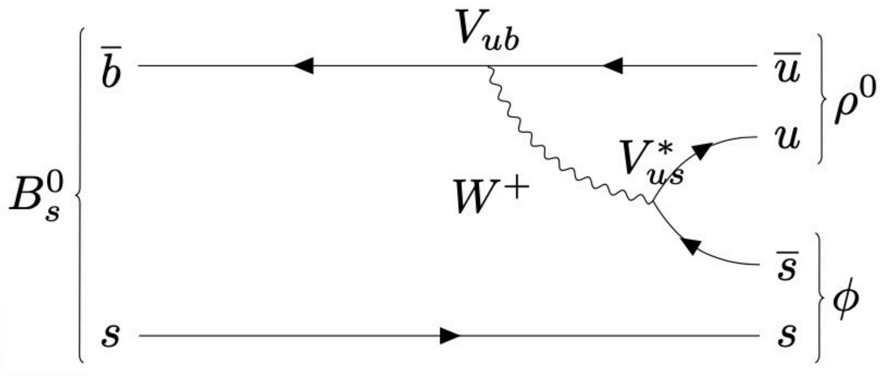


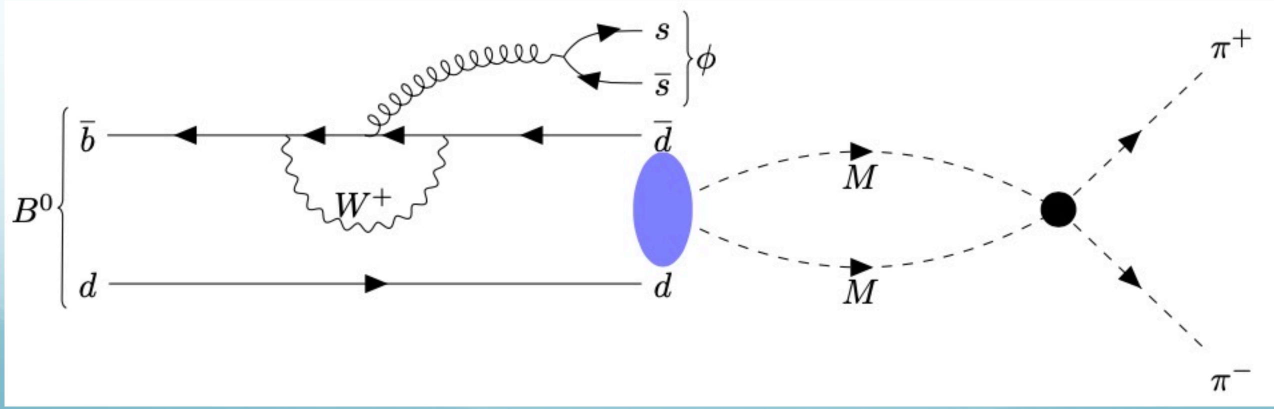


H. A. Ahmed and ***CWX***, Phys. Rev. D 101, 094034 (2020).

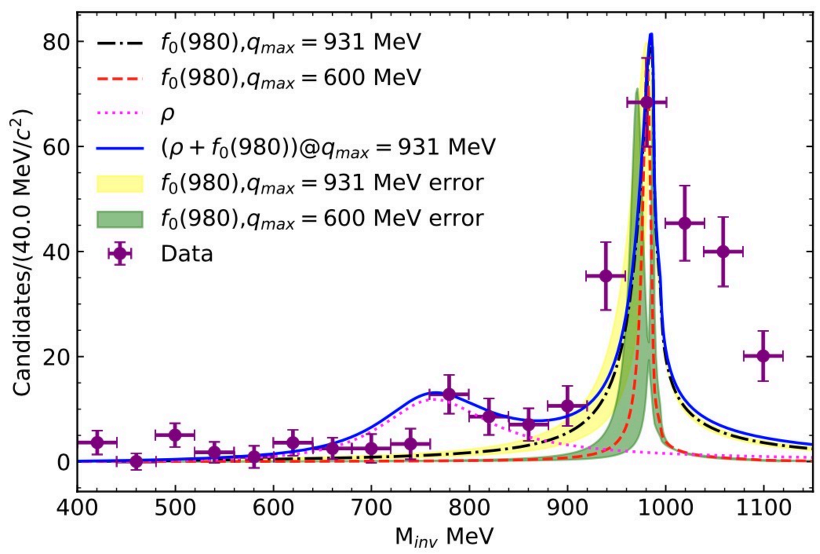
3) The results of the final state interaction



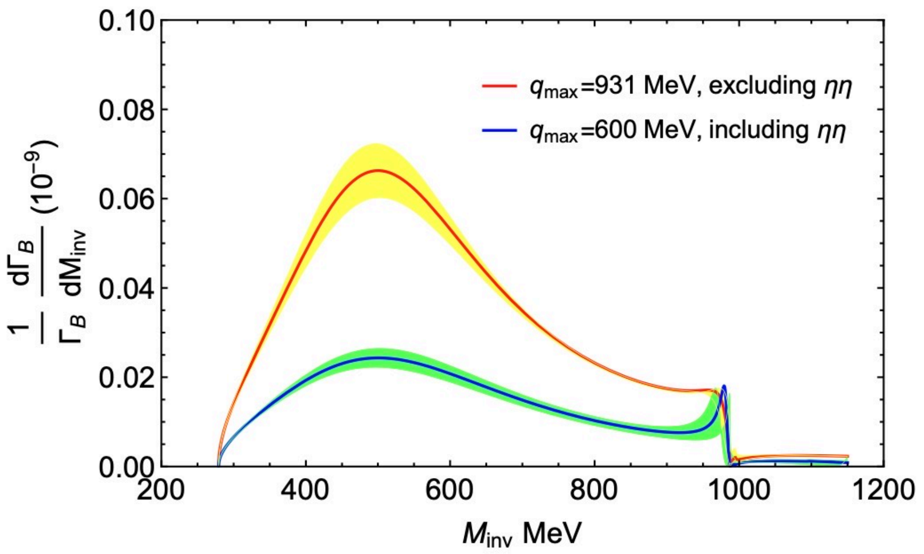




$B_s^0 \to \phi\pi^+\pi^-$



$B^0 \to \phi\pi^+\pi^-$



Branching ratios	Without $\eta\eta$ channel	With $\eta\eta$ channel	Exp.
$\mathrm{Br}(B^0 \to \phi f_0(980))$	$(5.21 \pm 0.98^{+4.40}_{-1.72}) \times 10^{-10}$	$(8.19 \pm 1.54^{+5.12}_{-2.34}) \times 10^{-10}$	$< 3.8 \times 10^{-7}$
$\mathrm{Br}(B^0 \to \phi f_0(500))$	$(6.89 \pm 1.29^{+0.27}_{-0.23}) \times 10^{-9}$	$(7.97 \pm 1.49^{+0.34}_{-0.30}) \times 10^{-9}$	-

Ratios	Without $\eta\eta$ channel	With $\eta\eta$ channel
$\frac{\mathrm{Br}(B^0 \to \phi f_0(980))}{\mathrm{Br}(B^0 \to J/\psi f_0(980))}$	$(9.28 \pm 3.05^{+0.26}_{-0.18}) \times 10^{-4}$	$(9.26 \pm 3.04^{+0.17}_{-0.15}) \times 10^{-4}$
$\frac{\mathrm{Br}(B^0 \to \phi f_0(500))}{\mathrm{Br}(B^0 \to J/\psi f_0(500))}$	$(7.87 \pm 2.58^{+0.03}_{-0.03}) \times 10^{-4}$	$(7.88 \pm 2.59^{+0.04}_{-0.03}) \times 10^{-4}$

$$R_1^{th}=\frac{\Gamma_{B^0\to\phi\rho^0}}{\Gamma_{B_s^0\to\phi\phi}}=\frac{1}{N_c^2}\frac{1}{4}\frac{1}{2}\left|\frac{V_{ub}V_{ud}+V_{cb}V_{cd}}{V_{ub}V_{us}+V_{cb}V_{cs}}\right|^2\frac{m_{B_s^0}^2}{m_{B^0}^2}\frac{p_{\rho^0}}{p_\phi}=6.70\times10^{-4},$$

$$R_2^{th}=\frac{\Gamma_{B^0\to\phi\omega}}{\Gamma_{B_s^0\to\phi\phi}}=\frac{1}{N_c^2}\frac{1}{4}\frac{1}{2}\left|\frac{V_{ub}V_{ud}+V_{cb}V_{cd}}{V_{ub}V_{us}+V_{cb}V_{cs}}\right|^2\frac{m_{B_s^0}^2}{m_{B^0}^2}\frac{p_{\omega}}{p_\phi}=6.70\times10^{-4},$$

$$R_3^{th}=\frac{\Gamma_{B_s^0\to\phi\bar{K}^{*0}}}{\Gamma_{B_s^0\to\phi\phi}}=\left|\frac{V_{ub}V_{ud}+V_{cb}V_{cd}}{V_{ub}V_{us}+V_{cb}V_{cs}}\right|^2\frac{p_{\bar{K}^{*0}}}{p_\phi}=4.72\times10^{-2}.$$

$$R_3^{exp}=\frac{\mathrm{Br}(B_s^0\to\phi\bar{K}^{*0})}{\mathrm{Br}(B_s^0\to\phi\phi)}=\frac{(1.14\pm0.30)\times10^{-6}}{(1.87\pm0.15)\times10^{-5}}=(6.09\pm2.09)\times10^{-2}$$

$$\mathrm{Br}(B^0\to\phi\rho^0)=\frac{\Gamma_{B^0\to\phi\rho^0}}{\Gamma_B}=(1.25\pm0.10)\times10^{-8},$$

$$\mathrm{Br}(B^0\to\phi\omega)=\frac{\Gamma_{B^0\to\phi\omega}}{\Gamma_B}=(1.25\pm0.10)\times10^{-8},$$

$$\mathrm{Br}(B_s^0\to\phi\bar{K}^{*0})=\frac{\Gamma_{B_s^0\to\phi\bar{K}^{*0}}}{\Gamma_{B_s}}=(8.83\pm0.71)\times10^{-7},$$

PDG

$$\mathrm{Br}(B^0\to\phi\rho^0)<3.3\times10^{-7},$$

$$\mathrm{Br}(B^0\to\phi\omega)<7\times10^{-7},$$

$$\mathrm{Br}(B_s^0\to\phi\bar{K}^{*0})=(1.14\pm0.30)\times10^{-6}.$$

H. A. Ahmed, Z. Y. Wang, Z. F. Sun and ***CWX***, arXiv: 2011.08758.

§4. Summary

- The $f_0(980)$ states is mainly a $K\bar{K}$ bound state.
- The σ state is a resonance of $\pi\pi$.
- The $a_0(980)$ state is a loose bound state of $K\bar{K}$, with the significant component of $\pi\eta$.
- These conclusions can be further confirmed in the final interaction results of $B^0_{(s)} \to \phi\pi^+\pi^-$ decays.

Hope that our predictions can be tested in the future experiments!

谢谢大家！

Thanks for your attention!