Exotic Mesons and Final State Interactions in Electron-Positron Collisions

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THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Phys. Lett. B 797, 134851 (2019)

Phys. Rev. D 102, 016019 (2020)

MESON 2021

1 Motivation & Goals

Motivation & Goals

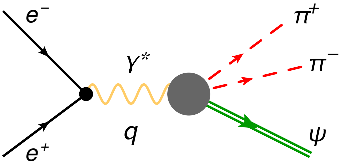
Charged Exotic States

- $e^+e^- \to \psi(2S)\,\pi^+\,\pi^-$
- $e^+e^- \to J/\psi\,\pi^+\,\pi^-$
- $e^+e^- \to h_c\,\pi^+\,\pi^-$

Goals

- Simultaneous description of the invariant mass distributions;
- $\pi\pi$ final state interaction using state-of-the-art dispersive formalism;
- Test hypothesis that charged exotic states are real resonances;
- Robust approach to obtain the quantum numbers of the intermediate states.

2 Formalism



$$s = (p_{\pi^+} + p_{\pi^-})^2$$
$$t = (p_\psi + p_{\pi^-})^2$$
$$u = (p_\psi + p_{\pi^+})^2$$

Double Differential Cross Section

$$\frac{\partial^2\sigma}{\partial s \partial t} = \frac{1}{3}\frac{e^2}{(2\pi)^3}\frac{1}{2^5}\frac{(q^2+2m_e^2)}{\sqrt{q^2(q^2-4m_e^2)}}\frac{1}{(q^2)^3}\sum_{\lambda_1\lambda_2}|\mathcal{H}_{\lambda_1\lambda_2}|^2$$

$$\langle \pi\pi\psi(\lambda_2)|\,\mathcal{T}\,|\gamma^*(\lambda_1)\rangle = (2\pi)^4\delta(q - p_\psi - p_{\pi^+} - p_{\pi^-})\ \mathcal{H}_{\lambda_1\lambda_2}$$

Independent Helicity Amplitudes

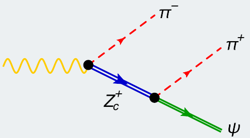
P-symmetry: $\mathcal{H}_{++}$, $\mathcal{H}_{+-}$, $\mathcal{H}_{+0}$, $\mathcal{H}_{0+}$ and $\mathcal{H}_{00}$

- CP-symmetry: $J_{\pi\pi} \to$ even
- Bose-symmetry: $I_{\pi\pi} = 0\ ,\ 2$
- $I_\psi = 0\ ;\ I_\gamma = 0\ ,\ 1 \implies \boxed{I_{\pi\pi} = 0}$

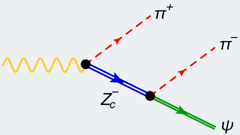
Rescattering s-channel

$$\mathcal{H}_{\lambda_1\lambda_2}(s,t) = H^{(Z_c)}_{\lambda_1\lambda_2}(s,t) + H^{R}_{\lambda_1\lambda_2}(s,t)$$

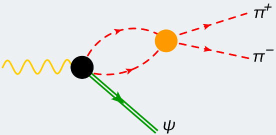
t-channel



u-channel



s-channel



Helicity Amplitude with Rescattering

$$\mathcal{H}_{\lambda_1\lambda_2}(s,t) = H^{(Z_c)}_{\lambda_1\lambda_2}(s,t) + \Omega(s)\left\{a + b\,s - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\,\frac{\mathrm{Disc}\left(\Omega^{-1}(s')\right)h^{L}_{\lambda_1\lambda_2}(s')}{s'-s}\right\}$$

- 2 subtraction constants to reduce the sensitive to high energy.

Left-Hand Cuts

Invariant Amplitudes

- The helicity amplitude $\mathcal{H}^{\mu\nu}$ can be written in the general form as

$$\mathcal{H}^{\mu\nu} = \sum_{i=1}^{5} F_i L_i^{\mu\nu}$$

where F_i are the invariant amplitudes and $L_i^{\mu\nu}$ is a complete set of Lorenz structures.

- For the S-wave:

$$h_{++}^{(0)}(s) = \frac{s - q^2 - m_\psi^2}{2} f_1(s) - q^2\, m_\psi^2 f_4(s)$$

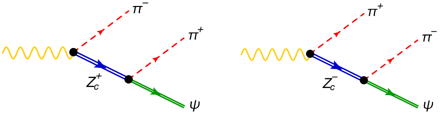
$$h_{00}^{(0)}(s) = -q m_\psi \left(f_1(s) - \frac{s - q^2 - m_\psi^2}{2}\, f_4(s) \right)$$

with f_i the partial wave expansion of F_i.

- The helicity amplitudes are correlated

$$h_{++}^{(0)}(s) \pm h_{00}^{(0)}(s) \sim \mathcal{O}(s - (q \pm m_\psi)^2)$$

Z_c Exchange Mechanism



$$\mathcal{H}_{\lambda_1\lambda_2}^{Z_c} = (V_{Z_\pm\psi\pi})^{\beta\nu}\, S_{\nu\mu}(Q_{z_\pm})\, (V_{\gamma^*\pi Z_\pm})^{\mu\alpha} \times \epsilon_\alpha(p_{\gamma^*}, \lambda_1)\, \epsilon_\beta^*(p_\psi, \lambda_2)$$

- We observe that

$$\mathcal{H}_{+-}^{Z_c} = \mathcal{H}_{+0}^{Z_c} = \mathcal{H}_{0+}^{Z_c} \approx 0$$

$$\sum_{\lambda_1\lambda_2} |\mathcal{H}_{\lambda_1\lambda_2}^{Z_c}|^2 = 2\,|\mathcal{H}_{++}^{Z_c}|^2 + |\mathcal{H}_{00}^{Z_c}|^2 \approx 3\,|\mathcal{H}_{++}^{Z_c}|^2$$

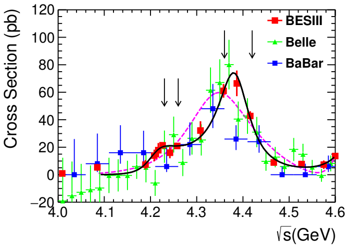
$\therefore$ the effects of the kinematical constraints can be ignored!

- pole contribution: $t = m_z^2$ and $u = m_z^2$ in the numerators.

3 $\psi(2S)\ \pi^+\pi^-$

$e^+e^- \to \psi(2S)\,\pi^+\pi^-$

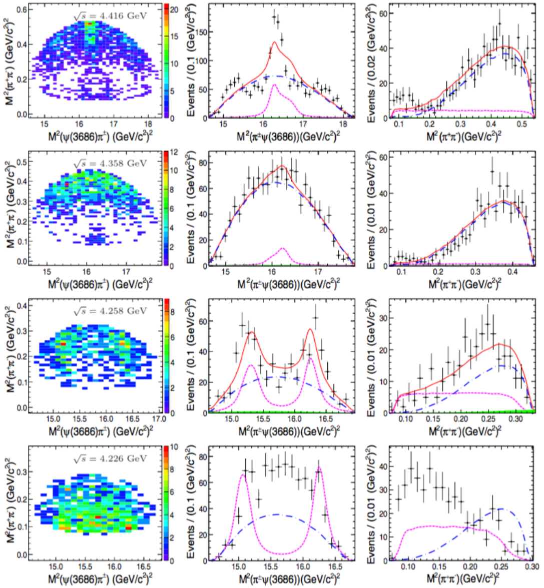
- **Y(4220) + Y(4390)**



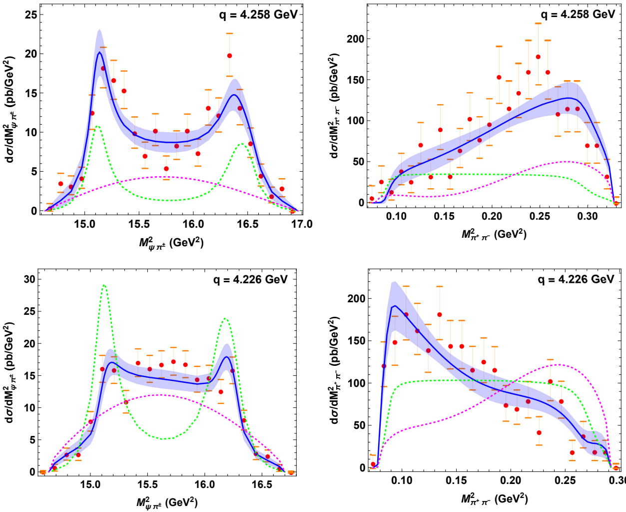
$Z_c(3900) + Z_c(4030)$

- Two charged exotic states!
- No consistent description
- Below $K\bar{K}$ threshold

BESIII PRD (2017)



$\mathrm{Z_c(3900)}$



$\chi^2_{red} = 1.01$

– Total
-- Only Z_c
-- Only $\pi\pi$-FSI

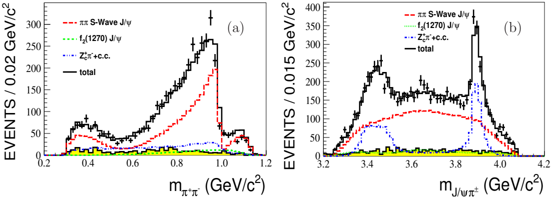
- $a(q^2)$ and $b(q^2)$ are complex numbers
- Global normalization: $N(q^2, \mathcal{F}_{\gamma^* Z_c \pi}, C_{Z_c \psi \pi})$

$\chi^2_{red} = 1.16$

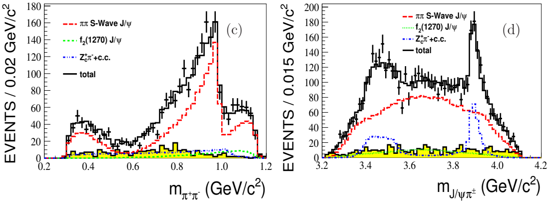
Data is normalized using the total cross section

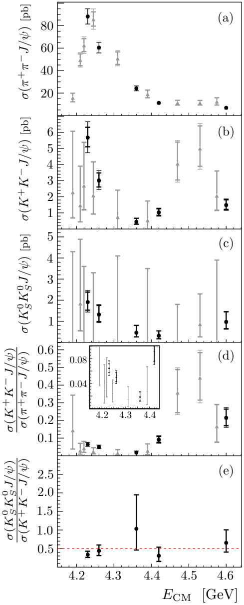
4 $J/\psi\ \pi^+\pi^-$

$e^+e^- \rightarrow J/\psi\ \pi^+\pi^-$



BESIII PRL (2017)





BESIII PRD (2018)

- Above $K\bar{K}$ threshold $\rightarrow$ couple channel rescattering;
- No evidence of the strange partner of $Zc(3900)$;
- $\sigma(J/\psi K^+K^-)$ is suppressed compared to $\sigma(J/\psi\ \pi^+\pi^-)$.

Strange Partner of $Z_c(3900)$?

$$\begin{bmatrix}\mathcal{H}_{\psi\pi\pi}\\ \mathcal{H}_{\psi KK}\end{bmatrix}=\begin{bmatrix}H^{Z_c}(s,t)\\ H^{Z_c^s}(s,t)\end{bmatrix}+\underbrace{\begin{bmatrix}\Omega_{\pi\pi,\pi\pi} & \Omega_{\pi\pi,K\bar{K}}\\ \Omega_{K\bar{K},\pi\pi} & \Omega_{K\bar{K},K\bar{K}}\end{bmatrix}}_{\bar{\Omega}}\left\{\begin{bmatrix}a+b\,s\\ c+d\,s\end{bmatrix}-\frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\,\frac{\mathrm{Disc}\,(\bar{\Omega})^{-1}(s')}{s'-s}\begin{bmatrix}h^{(0)}_{Z_c}(s')\\ h^{(0)}_{Z_c^s}(s')\end{bmatrix}\right\}$$

Strange Partner of $Z_c(3900)$?

$$\begin{bmatrix}\mathcal{H}_{\psi\pi\pi}\\ \mathcal{H}_{\psi KK}\end{bmatrix} = \begin{bmatrix}H^{Z_c}(s,t)\\ \cancel{H^{Z_c^s}}(s,t)\end{bmatrix} + \underbrace{\begin{bmatrix}\Omega_{\pi\pi,\pi\pi} & \Omega_{\pi\pi,K\bar{K}}\\ \Omega_{K\bar{K},\pi\pi} & \Omega_{K\bar{K},K\bar{K}}\end{bmatrix}}_{\bar{\Omega}} \left\{ \begin{bmatrix}a+b\,s\\ c+d\,s\end{bmatrix} - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{\mathrm{Disc}\,(\bar{\Omega})^{-1}(s')}{s'-s}\begin{bmatrix}h^{(0)}_{Z_c}(s')\\ \cancel{h^{(0)}_{Z_c^s}}(s')\end{bmatrix}\right\}$$

Naive Insights:

- $(m_s \approx 95$ MeV$) >> (m_u \approx 2.2$ MeV$)$ and $(m_d \approx 4.7$ MeV$)$

$$\Longrightarrow\ \mathbf{m_{Z_c^{(s)}} > m_{Z_c}}$$

- For $q = 4.26(4.23)$ GeV, $\gamma^*(q) \to K\ Z_c^{(s)}$ and $Z_c^{(s)} \to J/\psi\ K$ would constrain the $Z_c^{(s)}$ mass:

$$\mathbf{3.59 < m_{Z_c^{(s)}} < 3.77(3.74)\ GeV}$$

$Z_c^{(s)}(3985)$ recently observed in *BESIII PRL (2021)*

S- and D-wave Rescattering

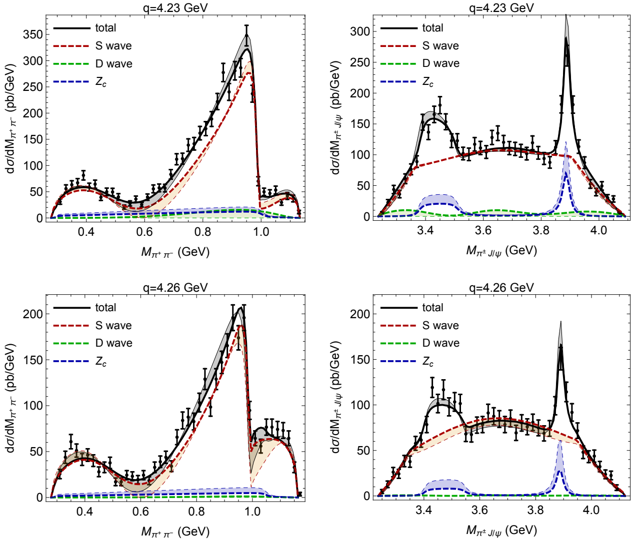
$$\begin{bmatrix}\mathcal{H}_{\psi\pi\pi}\\ \mathcal{H}_{\psi KK}\end{bmatrix}=\begin{bmatrix}H^{Z_c}(s,t)\\ \cancel{H^{Z_s}}(s,t)\end{bmatrix}+\underbrace{\begin{bmatrix}\Omega_{\pi\pi,\pi\pi} & \Omega_{\pi\pi,K\bar{K}}\\ \Omega_{K\bar{K},\pi\pi} & \Omega_{K\bar{K},K\bar{K}}\end{bmatrix}}_{\bar{\Omega}}\left\{\begin{bmatrix}a+b\,s\\ c+d\,s\end{bmatrix}-\frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{\mathrm{Disc}\,(\bar{\Omega})^{-1}(s')}{s'-s}\begin{bmatrix}h_{Z_c}^{(0)}(s')\\ \cancel{h_{Z_c}^{(0)}}(s')\end{bmatrix}\right\}$$

D-wave Rescattering:

$$\mathcal{H}^{(2)}_{\psi\pi\pi}=5P_2(z)\gamma(s)\Omega^{(2)}(s)\left\{e-\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\mathrm{Disc}\,(\Omega^{(2)})^{-1}(s')}{s'-s}\frac{h^{(2)}_{Z_c}(s')}{\gamma(s')}\right\}$$

Centrifugal Barrier Factor: $\gamma(s)\equiv(s-4m_\pi^2)(s-(q-m_\psi)^2)$

$e^+e^- \to J/\psi\ \pi^+\pi^-$



Fit 1: $\chi^2_{red} = 3.4$
Fit 2: $\chi^2_{red} = 1.7$

Extra Constrain:

$e^+e^- \to J/\psi\ K^+K^-$

- Global normalization: $N(q^2, \mathcal{F}_{\gamma^* Z_c \pi}, C_{Z_c \psi \pi})$

Fit 1 (4 pars)

a, b, d: real

Fit 2 (7 pars)

a, b: complex
d, e: real

Fit 1: $\chi^2_{red} = 2.5$
Fit 2: $\chi^2_{red} = 1.3$

5 Perspectives

Perspectives

$\mathrm{e^+e^-} \to \mathrm{h_c}\, \pi^+ \pi^-$ (Preliminary)

- Collaboration with BESIII;
- Extend the formalism for the h_c-case, with quantum number 1^{+-}; ☑
- Simultaneous description of $h_c\pi$ and $\pi\pi$ invariant mass distributions; ☑
- Not only $Z_c(4020)$, but also $Z_c(3900)$ as intermediate states; ☑
- Study of the possible quantum numbers of $Z_c(4020)$; □
- Determination of the mass and width of $Z_c(4020)$. □

6 Summary

Summary

$\mathbf{e^+e^- \rightarrow \psi(2S)\,\pi^+\,\pi^-}$

- The exotic state $Z_c(3900)$ plays an important role to explain the invariant mass distribution at both $q = 4.226$ and $q = 4.258$ GeV;
- The $\pi\pi$-FSI is the main mechanism to describe the $\pi\pi$-line shape for all the energies.

$\mathbf{e^+e^- \rightarrow J/\psi\,\pi^+\,\pi^-}$

- The exotic state $Z_c(3900)$ and the couple channel FSI are essential to describe the invariant mass distributions;
- The reactions $e^+e^- \rightarrow J/\psi\,\pi^+\pi^-$ and $e^+e^- \rightarrow J/\psi\,K^+K^-$ have to be analyzed simultaneously in order to constrain the parameters.

Thank you for listening!

SFB 1044
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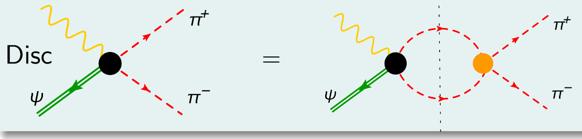
Appendix

Unitarity in the s-channel

$J_{\pi\pi} = 0$

$I_{\pi\pi} = 0$



$$\text{Disc}\, h^R_{\lambda_1\lambda_2}(s) \equiv \frac{1}{2\,i}(h_{\lambda_1\lambda_2}(s+i\epsilon) - h_{\lambda_1\lambda_2}(s-i\epsilon)) = t^*_{\pi\pi}(s)\;\rho(s)\;h_{\lambda_1\lambda_2}(s)\,\theta(s>4m^2_\pi)$$

- The pions interaction amplitude can be written in terms of the phase shift:

$$t^*_{\pi\pi}(s) = \frac{e^{-i\delta_{\pi\pi}(s)}\sin\delta_{\pi\pi}(s)}{\rho(s)}$$

- The $\pi\pi$-rescattering can be parametrized in terms of Ω: Disc $\Omega(s) = t^*_{\pi\pi}(s)\rho(s)\Omega(s)$
- Therefore, we can use the $\delta_{\pi\pi}(s)$ through the Omnès Function:

$$\Omega(s) = exp\left[\frac{s}{\pi}\int_{4m^2_\pi}^{\infty}\frac{ds'}{s'}\frac{\delta_{\pi\pi}(s')}{s'-s}\right]$$

Omnès, Nuovo Cim. (1958)
Muskhelishvili, (1953)

Dispersion Relation

- Assuming no kinematic constrains, we look for a solution in terms of the Omnès function:

$$h^R_{\lambda_1\lambda_2}(s) = \Omega(s)\; G_{\lambda_1\lambda_2}(s)$$

- The unitarity relation for the Omnès function is

$$\mathrm{Disc}\,\Omega(s) = t^*_{\pi\pi}(s)\,\rho(s)\,\Omega(s)\,\theta(s > 4m_\pi^2)$$

- Since $\mathrm{Disc}\,h_{\lambda_1\lambda_2}(s) = \mathrm{Disc}\,h^R_{\lambda_1\lambda_2}(s)$, one can write down a DR for $G_{\lambda_1\lambda_2}$:

$$G_{\lambda_1\lambda_2} = -\int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi}\,\frac{\mathrm{Disc}\,(\Omega^{-1}(s'))\,h^L_{\lambda_1\lambda_2}(s')}{s'-s}$$

Helicity Amplitude with Rescattering

$$\mathcal{H}_{\lambda_1\lambda_2}(s,t) = H^{(Z_c)}_{\lambda_1\lambda_2}(s,t) + \Omega(s)\left\{a + b\,s - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\,\frac{\mathrm{Disc}\,(\Omega^{-1}(s'))\,h^L_{\lambda_1\lambda_2}(s')}{s'-s}\right\}$$

- 2 subtraction constants to reduce the sensitive to high energy.

Anomalous Threshold

- Depending on the kinematics new nonphysical singularities might appear ($q^2 > 2m_\pi^2 + 2m_z^2 - m_\psi^2$).
- The anomalous piece that emerges because the anomalous branch point moves onto the first Riemann sheet distorting the integration contour. Effectively, that can be written as

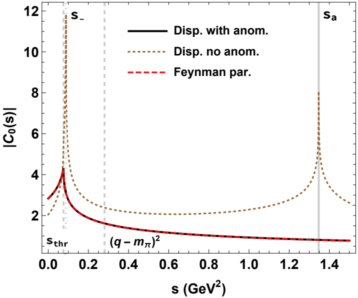
$$\int_{-1}^{1} \frac{dz}{t - m_z^2} = \int_{-1}^{1} \frac{dz}{u - m_z^2} = -\frac{2}{k(s)} log\left(\frac{X(s)+1}{X(s)-1}\right) - i\frac{4\pi}{k(s)}\theta(s_- < s < s_a)$$

- $s_a = 2m_\pi^2 + m_\psi^2 + q^2 - 2m_z^2$ is the position where the argument of the logarithm changes sign.
- Analytical continuation: $q \to q + i\epsilon$

Cross-Check

- Comparison of the DR with the scalar triangle loop calculated via traditional method.

S. Mandelstam, PRL (1960); W. Lucha *et al*, PRD (2007); M. Hoferichter *et al*, Mod. Phys. Conf. Ser. (2014)



$\pi\pi$ FSI: Single Channel

Modified-IAM for (J=0):

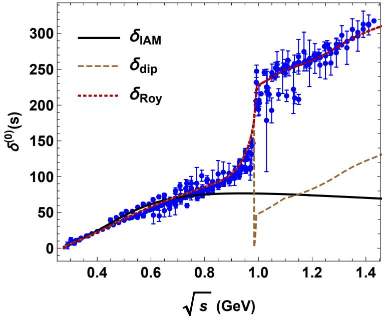
$$t_{\pi\pi}(s) = \frac{|t_{\rm LO}(s)|^2}{t_{\rm LO}(s) - t_{\rm NLO}(s) + A^{\rm mIAM}(s)}$$

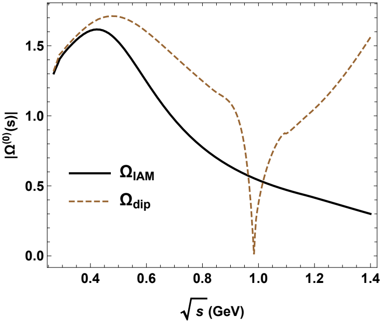
- Correct positions of Adler zeros;
- Consistent description of $f_0(500)$.

GomezNicola et al. PRD (2008)

Omnès Function

$$\Omega(s) = exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\delta_{\pi\pi}(s')}{s'-s}\right]$$





Inverse Amplitude Method

- partial wave elastic unitarity

$$\mathrm{Im}[t_{\pi\pi}(s)] = \rho_{\pi\pi}(s)\, |t_{\pi\pi}(s)|^2 \implies \mathrm{Im}\left[\frac{1}{t_{\pi\pi}(s)}\right] = -\rho_{\pi\pi}(s)$$

- The ChPT amplitude only satisfy the unitarity condition perturbatively:

$$\mathrm{Im}[t_{\mathrm{LO}}] = 0 \;; \qquad \mathrm{Im}[t_{\mathrm{NLO}}] = \rho_{\pi\pi}(s)\, |t_{\mathrm{LO}}(s)|^2 \qquad \text{with } t_{\pi\pi} = t_{\mathrm{LO}} + t_{\mathrm{NLO}} + \cdots$$

- One can write down a dispersion relation for the ChPT amplitudes as

$$t_{\mathrm{LO}}(s) = \sum_{l=0}^{k} a_l s^l \;; \qquad t_{\mathrm{NLO}}(s) = \sum_{l=0}^{k} b_l s^l + \frac{s^k}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^k} \frac{\mathrm{Im}\,[t_{\mathrm{NLO}}]}{s' - s - i\epsilon} + I_{LC}\,[t_{\mathrm{NLO}}]$$

- Same analytic structure for t and t^{-1}: $G(s) \equiv t_{\mathrm{LO}}^2 / t_{\pi\pi} \implies \mathrm{Im}[G(s)] = -\mathrm{Im}[t_{\mathrm{NLO}}]$
- Thus $G(s) \simeq t_{\mathrm{LO}}(s) - t_{\mathrm{NLO}}(s)$, considering that $I_{LC}\,[G(s)] = -I_{LC}\,[t_{\mathrm{NLO}}]$ and ignoring pole contributions:

$$\boxed{t_{\pi\pi}(s) \simeq \frac{|t_{\mathrm{LO}}(s)|^2}{t_{\mathrm{LO}}(s) - t_{\mathrm{NLO}}(s)}}$$

Truong, PRL (1988)
Dobado & Peláez, PRD (1993)

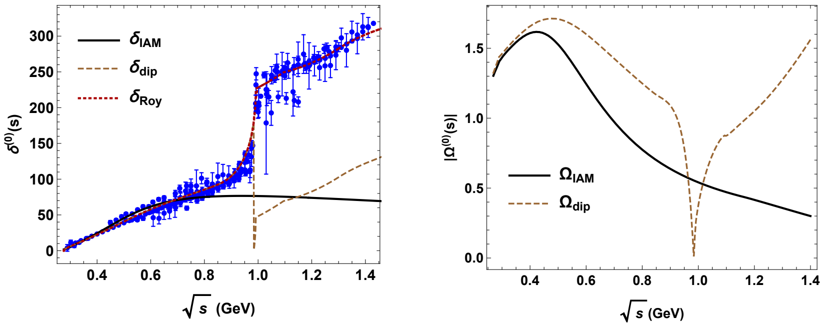
Inverse Amplitude Method

- Spurious poles emerges below threshold for the scalar waves (J=0), thus to reproduce correctly the Adlers zeros the IAM must be modified as

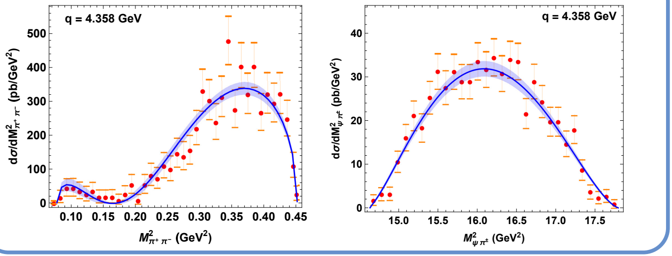
$$t_{\pi\pi}(s) = \frac{|t_{\mathrm{LO}}(s)|^2}{t_{\mathrm{LO}}(s) - t_{\mathrm{NLO}}(s) + A^{\mathrm{mIAM}}(s)}$$

GomezNicola et al. PRD (2008)

with the adler zero $s_A = s_{LO} + s_{NLO} + \mathcal{O}(p^6)$ and $t_{\mathrm{LO}}(s_{\mathrm{LO}} + s_{\mathrm{NLO}}) + t_{\mathrm{NLO}}(s_{\mathrm{LO}} + s_{\mathrm{NLO}}) = 0$



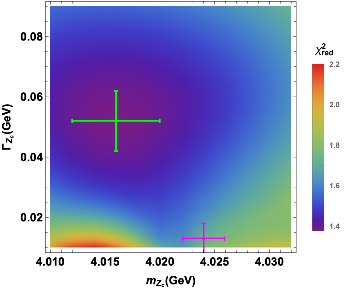
No Z_c Intermediate State



$\chi^2_{red} = 0.83$

- No intermediate state is required;
- Two real subtraction constants and the $\pi\pi$ Omnès function describe well the data;
- The left-hand cuts are dominated by the contact interation or possible D-meson loops (absorbed in the subtraction constants).

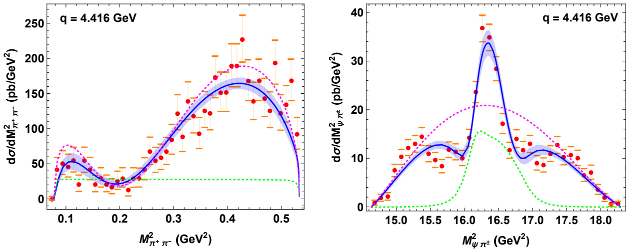
$Z_c(4016)$ or $Z_c(4020)$?



m_{Z_c} (GeV)	Γ_{Z_c} (MeV)
4.016(4)	52(10)
4.024(2)	13(5)

$Z_c(4020)$ observed by BESIII in

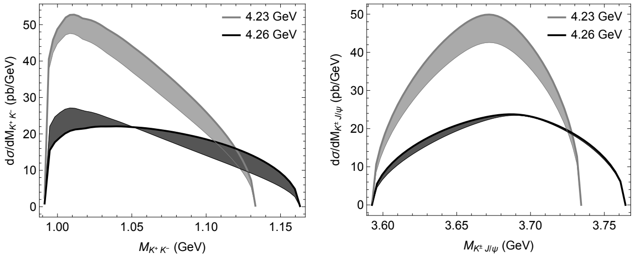
- $e^+e^- \to D^*\bar{D}^*\pi$
- $e^+e^- \to h_c\pi\pi$



$\chi^2_{red} = 1.38$

– Total
-- Only Z_c
-- Only $\pi\pi$-FSI

Predictions for $e^+e^- \to J/\psi\, K^+K^-$



$\sigma^{\text{fit 1}}_{4.23} = 4.4 \pm 0.5$ pb

$\sigma^{\text{fit 2}}_{4.23} = 5.2 \pm 0.2$ pb

$\sigma^{\text{BES}}_{4.23} = 5.3 \pm 1.0$ pb

$\sigma^{\text{fit 1}}_{4.26} = 2.9 \pm 0.4$ pb

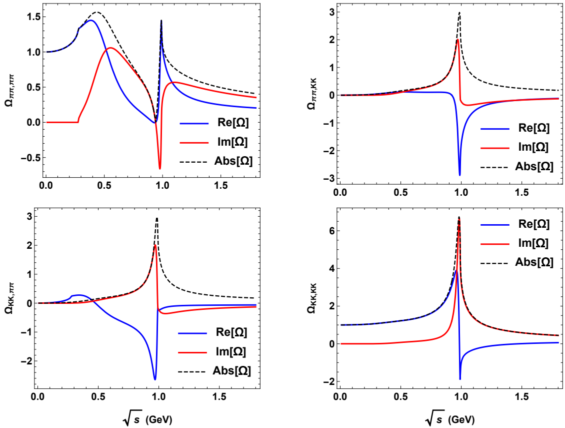
$\sigma^{\text{fit 2}}_{4.26} = 3.0 \pm 0.3$ pb

$\sigma^{\text{BES}}_{4.26} = 3.1 \pm 0.6$ pb

- The experimental cross section σ^{BES} is used to constrain the fits;
- K^+K^- predictions are given by $K\bar{K}, \pi\pi$ and $K\bar{K}, K\bar{K}$ final state interactions.

$\pi\pi, \bar{K}K$ FSI: Couple Channel

$$\begin{bmatrix} \mathcal{H}_{\psi\pi\pi} \\ \mathcal{H}_{\psi KK} \end{bmatrix} = \begin{bmatrix} H^{Z_c}(s,t) \\ H^{Z_c^s}(s,t) \end{bmatrix} + \underbrace{\begin{bmatrix} \Omega_{\pi\pi,\pi\pi} & \Omega_{\pi\pi,K\bar{K}} \\ \Omega_{K\bar{K},\pi\pi} & \Omega_{K\bar{K},K\bar{K}} \end{bmatrix}}_{\bar{\Omega}} \left\{ \begin{bmatrix} a + b\,s \\ c + d\,s \end{bmatrix} - \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{\mathrm{Disc}\,(\bar{\Omega})^{-1}(s')}{s' - s} \begin{bmatrix} h^L_{Z_c}(s') \\ h^L_{Z_c^s}(s') \end{bmatrix} \right\}$$



Danilkin et al., arXiv:2012.11636