# Modification of heavy mesons in a hot medium within effective hadronic theories


Glòria Montaña

University of Barcelona
Institute of Cosmos Sciences


[GM, Angels Ramos, Laura Tolos, Juan Torres-Rincon, Phys.Lett.B 806 (2020)]
[GM, Angels Ramos, Laura Tolos, Juan Torres-Rincon, Phys.Rev.D 102 (2020)]
[GM, Olaf Kaczmarek, Laura Tolos, Angels Ramos, Eur.Phys.J.A 56 (2020)]

16th International Workshop on Meson Physics
17th - 20th May 2021

# Motivation

# MOTIVATION

- Matter at very high temperatures and vanishing baryon densities (QGP?) is produced in HICs at RHIC and LHC $\longrightarrow$ hot mesonic (pionic) matter
- Due to the large mass and relaxation time of the $c$ quark, charmed mesons are a powerful probe of the QGP $\longrightarrow$ quarkonia suppression: color screening + comover scattering
- Properties of hadrons and their thermal modification are contained in their spectral functions
- Spectral functions can be directly calculated using effective hadronic theories within a unitarized approach

# Scattering of open heavy-flavour mesons off light mesons in free space

# EFFECTIVE THEORY

Lagrangian at NLO in the chiral expansion and LO in the heavy-quark expansion

$$\mathcal{L}(D^{(*)},\Phi) = \mathcal{L}_{\rm LO}(D^{(*)},\Phi) + \mathcal{L}_{\rm NLO}(D^{(*)},\Phi)$$

$$\mathcal{L}_{\rm LO} = \langle \nabla^\mu D \nabla_\mu D^\dagger \rangle - m_D^2 \langle D D^\dagger \rangle - \langle \nabla^\mu D^{*\nu} \nabla_\mu D^{*\dagger}_\nu \rangle + m_D^2 \langle D^{*\nu} D^{*\dagger}_\nu \rangle$$
$$+ i\, g \langle D^{*\mu} u_\mu D^\dagger - D u^\mu D^{*\dagger}_\mu \rangle + \frac{g}{2m_D} \langle D^*_\mu u_\alpha \nabla_\beta D^{*\dagger}_\nu - \nabla_\beta D^*_\mu u_\alpha D^{*\dagger}_\nu \rangle \epsilon^{\mu\nu\alpha\beta}$$

[Kolomeitsev and Lutz (2004)]
[Lutz and Soyeur (2008)]
[Guo, Hanhart and Meißner (2009)]
[Geng, Kaiser, Martin-Camalich and Weise (2010)]

$D$ mesons:

$$D = \begin{pmatrix} D^0 & D^+ & D_s^+ \end{pmatrix},$$
$$D^*_\mu = \begin{pmatrix} D^{*0} & D^{*+} & D_s^{*+} \end{pmatrix}_\mu$$

Light mesons:

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

# EFFECTIVE THEORY

Lagrangian at NLO in the chiral expansion and LO in the heavy-quark expansion

$$\mathcal{L}(D^{(*)},\Phi) = \mathcal{L}_{\rm LO}(D^{(*)},\Phi) + \mathcal{L}_{\rm NLO}(D^{(*)},\Phi)$$

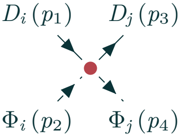
$$\begin{aligned}\mathcal{L}_{\rm LO} = \; & \langle \nabla^\mu D \nabla_\mu D^\dagger \rangle - m_D^2 \langle D D^\dagger \rangle \; - \langle \nabla^\mu D^{*\nu} \nabla_\mu D^{*\dagger}_\nu \rangle + m_D^2 \langle D^{*\nu} D^{*\dagger}_\nu \rangle \\ & + i\, g \langle D^{*\mu} u_\mu D^\dagger - D u^\mu D^{*\dagger}_\mu \rangle + \frac{g}{2 m_D} \langle D^*_\mu u_\alpha \nabla_\beta D^{*\dagger}_\nu - \nabla_\beta D^*_\mu u_\alpha D^{*\dagger}_\nu \rangle \epsilon^{\mu\nu\alpha\beta}\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\rm NLO} = & - \; h_0 \, \langle D D^\dagger \rangle \langle \chi_+ \rangle + \; h_1 \, \langle D \chi_+ D^\dagger \rangle + \; h_2 \, \langle D D^\dagger \rangle \langle u^\mu u_\mu \rangle \\ & + \; h_3 \, \langle D u^\mu u_\mu D^\dagger \rangle + \; h_4 \, \langle \nabla_\mu D \nabla_\nu D^\dagger \rangle \langle u^\mu u^\nu \rangle + \; h_5 \, \langle \nabla_\mu D \{u^\mu, u^\nu\} \nabla_\nu D^\dagger \rangle \\ & + \; \tilde{h}_0 \, \langle D^{*\mu} D^{*\dagger}_\mu \rangle \langle \chi_+ \rangle - \; \tilde{h}_1 \, \langle D^{*\mu} \chi_+ D^{*\dagger}_\mu \rangle - \; \tilde{h}_2 \, \langle D^{*\mu} D^{*\dagger}_\mu \rangle \langle u^\nu u_\nu \rangle \\ & - \; \tilde{h}_3 \, \langle D^{*\mu} u^\nu u_\nu D^{*\dagger}_\mu \rangle - \; \tilde{h}_4 \, \langle \nabla_\mu D^{*\alpha} \nabla_\nu D^{*\dagger}_\alpha \rangle \langle u^\mu u^\nu \rangle - \; \tilde{h}_5 \, \langle \nabla_\mu D^{*\alpha} \{u^\mu, u^\nu\} \nabla_\nu D^{*\dagger}_\alpha \rangle\end{aligned}$$

$$\text{LECs}: \; h_{0,\dots,5}, \; \tilde{h}_{0,\dots,5}$$

[Liu, Orginos, Guo, Hanhart and Meißner (2013)]
[Tolos and Torres-Rincon (2013)]
[Albaladejo, Fernandez-Soler, Guo and Nieves (2017)]
[ Guo, Liu, Meißner, Oller and Rusetsky (2019)]

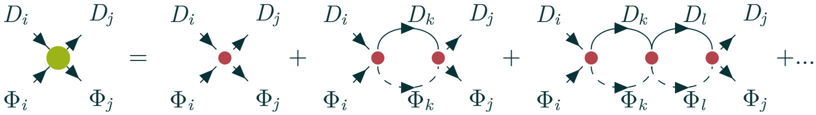
# SCATTERING IN COUPLED CHANNELS




s-wave scattering amplitude of $D^{(*)}$, $D_s^{(*)}$ mesons with $\pi$, $K$, $\bar{K}$, $\eta$ mesons:

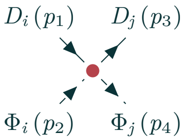
$$\mathcal{L} \to V^{ij}(s)$$

## Unitarization: Bethe-Salpeter equation




$$T_{ij} = V_{ij} + V_{ik} G_k V_{kj} + V_{ik} G_k V_{kl} G_l V_{lj} + \ldots$$

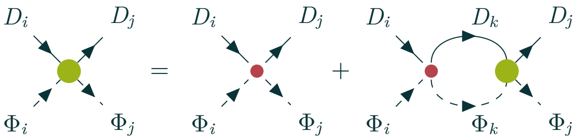
# SCATTERING IN COUPLED CHANNELS




s-wave scattering amplitude of $D^{(*)}$, $D_s^{(*)}$ mesons with $\pi$, $K$, $\bar{K}$, $\eta$ mesons:

$$\mathcal{L} \to V^{ij}(s)$$

## Unitarization: Bethe-Salpeter equation




$\longrightarrow$

$$T_{ij} = V_{ij} + V_{ik} G_k T_{kj}$$

On-shell factorization of the $T$-matrix:

$$T = (1 - VG)^{-1} V$$

- The two-meson propagator is regularized with a cutoff

$$G_k = i \int^{\Lambda} \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_{D,k}^2 + i\varepsilon} \frac{1}{(P-q)^2 - m_{\Phi,k}^2 + i\varepsilon}$$

- Poles in different Riemann sheets $\longrightarrow$ bound staes, resonances and virtual states.
- Identification of the dynamically generated states with the experimental $D_0^*(2300)$, $D_{s0}^*(2317)$, $D_1^*(2430)$ and $D_{s1}^*(2460)$.

# RESULTS: DYNAMICALLY GENERATED STATES

| | $D_0^*(2300)$ | $D_{s0}^*(2317)$ | $D_1^*(2430)$ | $D_{s1}^*(2460)$ |
|---|---|---|---|---|
| $M_R$ (MeV) | $2300 \pm 19$ | $2317.8 \pm 0.5$ | $2427 \pm 40$ | $2459.5 \pm 0.6$ |
| $\Gamma_R$ (MeV) | $274 \pm 40$ | $< 3.8$ | $384^{+130}_{-110}$ | $< 3.5$ |

| $J^P$ | $(S, I)$ | Coupled channels | | | RS | Poles (MeV) | Couplings (GeV) |
|---|---|---|---|---|---|---|---|
| $0^+$ | $(0, \frac{1}{2})$ | $D\pi$ | $D\eta$ | $D_s\bar{K}$ | $(-,+,+)$ | $2081.9 - i86.0$ | $\lvert g_{D\pi}\rvert = 8.9$ , $\lvert g_{D\eta}\rvert = 0.4$, $\lvert g_{D_s\bar{K}}\rvert = 5.4$ |
| | | $(2005.28)$ | $(2415.10)$ | $(2463.98)$ | $(-,-,+)$ | $2529.3 - i145.4$ | $\lvert g_{D\pi}\rvert = 6.7$, $\lvert g_{D\eta}\rvert = 9.9$, $\lvert g_{D_s\bar{K}}\rvert = 19.4$ |
| | $(1, 0)$ | $DK$ | $D_s\eta$ | | $(+,+)$ | $2252.5 - i0.0$ | $\lvert g_{DK}\rvert = 13.3$ , $\lvert g_{D_s\eta}\rvert = 9.2$ |
| | | $(2364.88)$ | $(2516.20)$ | | | | |
| $1^+$ | $(0, \frac{1}{2})$ | $D^*\pi$ | $D^*\eta$ | $D_s^*\bar{K}$ | $(-,+,+)$ | $2222.3 - i84.7$ | $\lvert g_{D^*\pi}\rvert = 9.5$ , $\lvert g_{D^*\eta}\rvert = 0.4$, $\lvert g_{D_s^*\bar{K}}\rvert = 5.7$ |
| | | $(2146.59)$ | $(2556.42)$ | $(2607.84)$ | $(-,-,+)$ | $2654.6 - i117.3$ | $\lvert g_{D^*\pi}\rvert = 6.5$, $\lvert g_{D^*\eta}\rvert = 10.0$, $\lvert g_{D_s^*\bar{K}}\rvert = 18.5$ |
| | $(1, 0)$ | $D^*K$ | $D_s^*\eta$ | | $(+,+)$ | $2393.3 - i0.0$ | $\lvert g_{D^*K}\rvert = 14.2$ , $\lvert g_{D_s^*\eta}\rvert = 9.7$ |
| | | $(2504.20)$ | $(2660.06)$ | | | | |

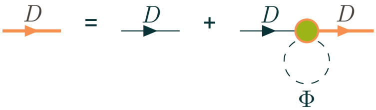
# Thermal corrections

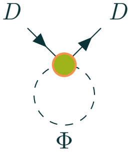
# THERMAL MODIFICATION OF HEAVY MESONS IN A MESONIC BATH

- Imaginary-time formalism
  - Sum over Matsubara frequencies $\rightarrow$ Bose-Einstein distribution functions

$$q^0 \rightarrow i\omega_n = \frac{i}{\beta} 2\pi n, \quad \int \frac{d^4 q}{(2\pi^4)} \rightarrow \frac{i}{\beta} \sum_n \int \frac{d^3 q}{(2\pi)^3} \quad \text{(bosons)}$$

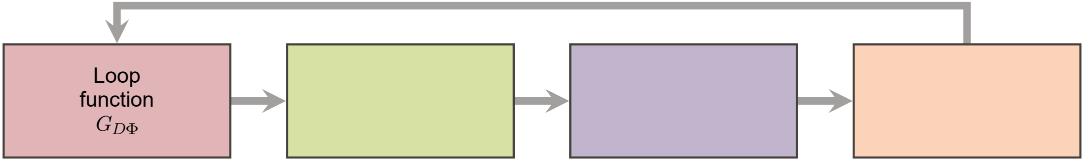
- Dressing the mesons in the loop function
  - Self-energy corrections
  - Pion mass varies slightly below $T_c$ $\rightarrow$ only the heavy meson is dressed




In the bath, processes that are forbidden in free space become possible: both production and absorption of heavy-light pairs.

# SELF-CONSISTENT ITERATIVE PROCEDURE



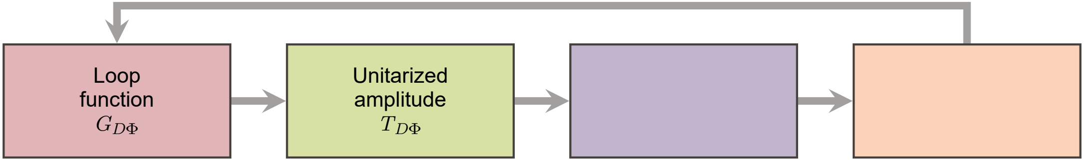

$$G_{D\Phi}(E,\vec{p};T) = \int \frac{d^3 q}{(2\pi)^3} \int d\omega \int d\omega' \, \frac{S_D(\omega,\vec{q};T) S_\Phi(\omega',\vec{p}-\vec{q};T)}{E-\omega-\omega'+i\varepsilon} [1 + f(\omega,T) + f(\omega',T)]$$

Spectral functions

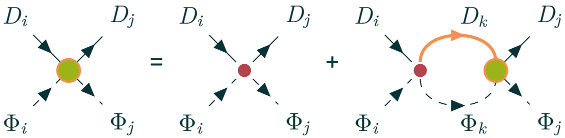
Bose distribution function at T: $f(\omega, T) = \frac{1}{e^{\omega/T}-1}$ (At zero temperature $f(\omega, T=0) = 0$ .)

Regularized with a cutoff $\Lambda$

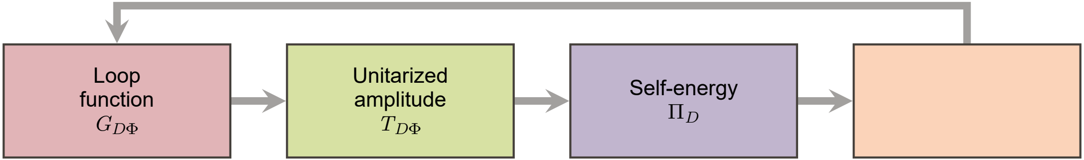
# SELF-CONSISTENT ITERATIVE PROCEDURE



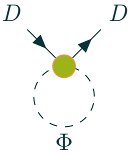

$$T_{ij} = V_{ij} + V_{ik} G_k T_{kj}$$



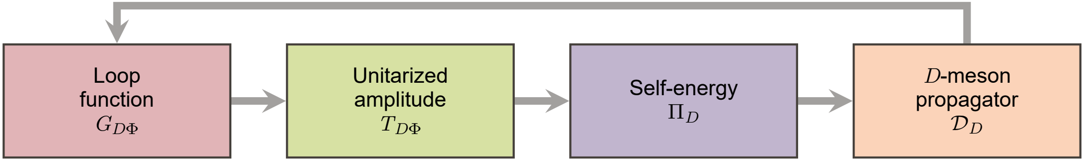
# SELF-CONSISTENT ITERATIVE PROCEDURE



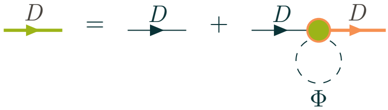

$$\Pi_D(E,\vec{p};T) = \frac{1}{\pi}\int \frac{d^3q}{(2\pi)^3}\int d\Omega \frac{E}{\omega_\Phi}\frac{f(\Omega,T) - f(\omega_\Phi,T)}{E^2-(\omega_\Phi-\Omega)^2+i\varepsilon}\,\mathrm{Im}\,T_{D\Phi}(\Omega,\vec{p}+\vec{q};T)$$



# SELF-CONSISTENT ITERATIVE PROCEDURE




$$S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \mathrm{Im}\, \mathcal{D}_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \mathrm{Im} \left( \frac{1}{\omega^2 - \vec{q}^{\,2} - m_D^2 - \Pi_D(\omega, \vec{q}; T)} \right)$$



# Results: Thermal modification of open-charm mesons

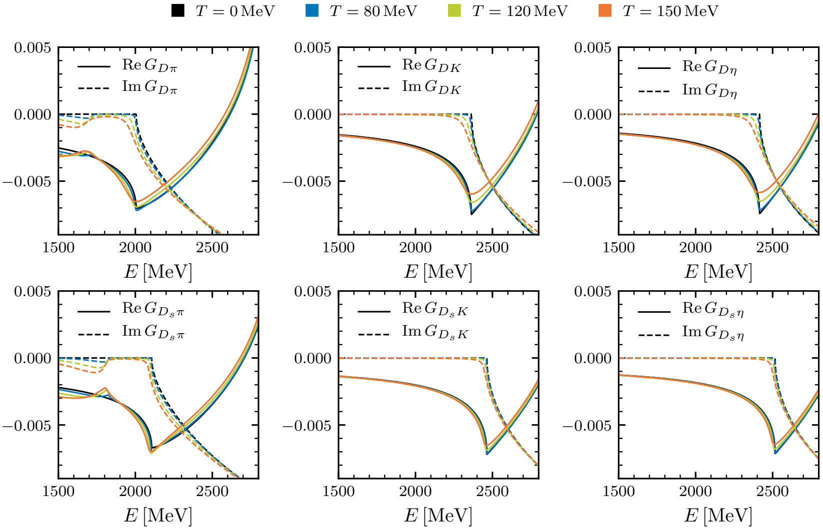
# LOOP FUNCTIONS

Pionic bath

$D$ and $D_s$ with light mesons

Unitary cut:
$E \geq (m_D + m_\Phi)$

Landau cut:
$E \leq (m_D - m_\Phi)$



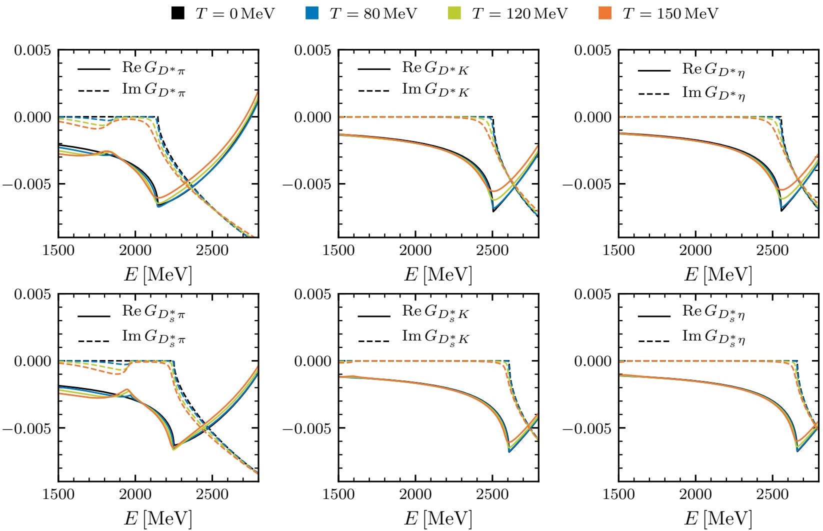
# LOOP FUNCTIONS

Pionic bath

$D^*$ and $D_s^*$ with light mesons

Unitary cut:
$E \geq (m_D + m_\Phi)$

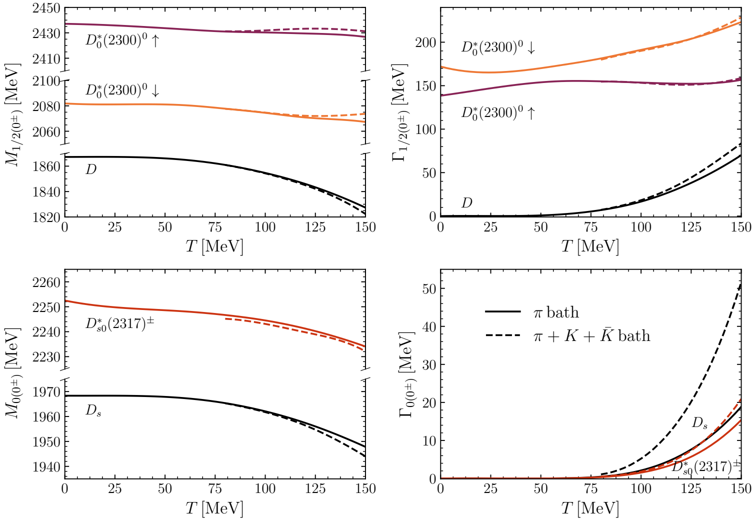
Landau cut:
$E \leq (m_D - m_\Phi)$



# CHIRAL PARTNERS

Evolution of masses and widths of the open-charm chiral partners in a pionic (or $\pi + K + \bar{K}$) bath

$$I(J^P) = \frac{1}{2}(0^\pm),\ 0(0^\pm)$$

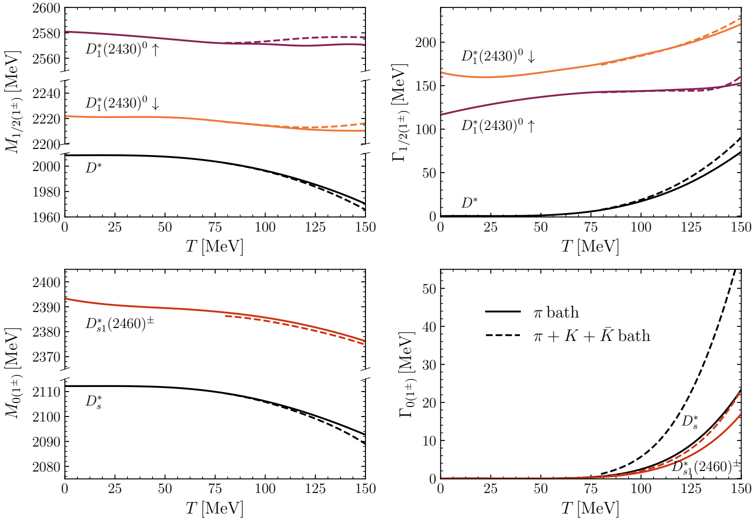



[GM, A. Ramos, L. Tolos, J. Torres-Rincon, Phys.Rev.D 102 (2020)]

# CHIRAL PARTNERS

Evolution of masses and widths of the open-charm chiral partners in a pionic (or $\pi + K + \bar{K}$) bath

$$I(J^P) = \frac{1}{2}(1^\pm),\ 0(1^\pm)$$




[GM, A. Ramos, L. Tolos, J. Torres-Rincon, Phys.Rev.D 102 (2020)]

# Euclidean correlators: comparison with lattice QCD

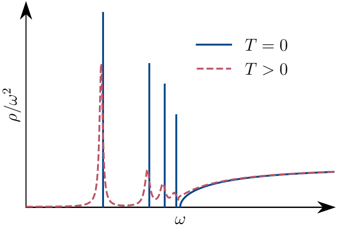
# FROM SPECTRAL FUNCTIONS TO EUCLIDEAN CORRELATORS

Spectral function $\rho(\omega, \vec{p}; T) \longrightarrow$ Euclidean correlator $G_E(\tau, \vec{p}; T)$

$$G_E(\tau, \vec{p}; T) = \int_0^\infty d\omega \; K(\tau, \omega; T) \; \rho(\omega, \vec{p}; T) \quad \rightarrow \quad K(\tau, \omega; T) = \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh(\frac{\omega}{2T})}$$

Euclidean correlator $\longrightarrow$ Spectral function (ill-posed)

- Bayesian methods (e.g. MEM)
- Fitting Ansätze




$$S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \mathrm{Im} \left( \frac{1}{\omega^2 - \vec{q}^2 - M_D^2 - \Pi_D(\omega, \vec{q}; T)} \right)$$

at unphysical meson masses (used in the lattice)

- Full: $\rho(\omega; T) = \rho_{\mathrm{gs}}(\omega; T) + a\rho_{\mathrm{cont}}(\omega; T)$

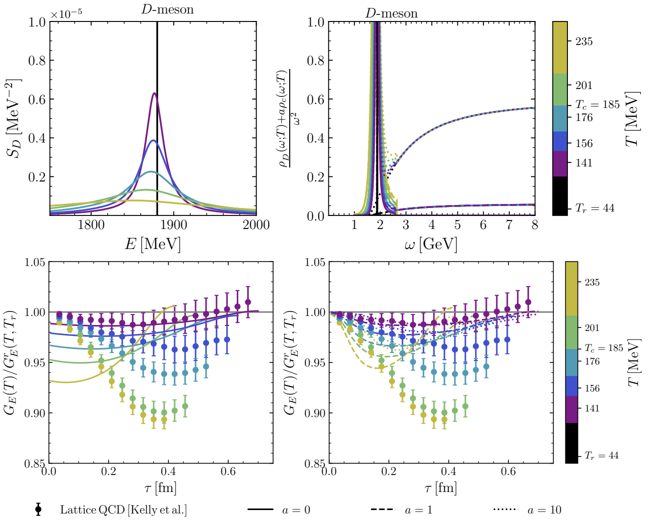
# EUCLIDEAN CORRELATORS WITH EFT

[GM, O. Kaczmarek, L. Tolos, A. Ramos, Eur.Phys.J.A 56 (2020)]

$m_\pi = 384$ MeV
$m_K = 546$ MeV
$m_\eta = 589$ MeV
$m_D = 1880$ MeV
$m_{D_s} = 1943$ MeV

[Kelly, Rothkopf, Skullerud (2018)]

- The inclusion of the continuum improves the comparison at small $\tau$
- Good agreement at the lowest temperature. At larger temperatures: excited states?
- Close and above $T_c$ the EFT breaks down
- Similiar results for the $D_s$



# Transport coefficients of an off-shell $D$ meson

# TRANSPORT COEFFICIENTS OF AN OFF-SHELL $D$-MESON

## Fokker-Planck equation for the Green's function

$$\frac{\partial}{\partial t} G_D^<(t,k) = \frac{\partial}{\partial k^i}\left\{ \hat{A}(k;T)\ k^i G_D^<(t,k) + \frac{\partial}{\partial k^j}\left[ \hat{B}_0(k;T)\ \Delta^{ij} + \hat{B}_1(k;T)\ \frac{k^i k^j}{\mathbf{k}^2}\right] G_D^<(t,k)\right\}$$

## Off-shell transport coefficients

- Drag force

$$\hat{A}(k^0,\mathbf{k};T) \equiv \frac{1}{2k^0}\int \frac{dk_1^0}{2\pi}\frac{d^3q}{(2\pi)^3} W(k^0,\mathbf{k},k_1^0,\mathbf{q})\ \frac{\mathbf{q}\cdot\mathbf{k}}{\mathbf{k}^2}$$

- Diffusion coefficients

$$\hat{B}_0(k^0,\mathbf{k};T) \equiv \frac{1}{4}\frac{1}{2k^0}\int \frac{dk_1^0}{2\pi}\frac{d^3q}{(2\pi)^3} W(k^0,\mathbf{k},k_1^0,\mathbf{q})\ \left[\mathbf{q}^2 - \frac{(\mathbf{q}\cdot\mathbf{k})^2}{\mathbf{k}^2}\right]$$

$$\hat{B}_1(k^0,\mathbf{k};T) \equiv \frac{1}{2}\frac{1}{2k^0}\int \frac{dk_1^0}{2\pi}\frac{d^3q}{(2\pi)^3} W(k^0,\mathbf{k},k_1^0,\mathbf{q})\ \frac{(\mathbf{q}\cdot\mathbf{k})^2}{\mathbf{k}^2}$$

$$\frac{1}{2k^0}\int \frac{dk_1^0}{2\pi}\frac{d^3q}{(2\pi)^3} W(k^0,\mathbf{k},k_1^0,\mathbf{q}) = \frac{1}{2k^0}\sum_{\lambda,\lambda'=\pm}\lambda\lambda'\int_{-\infty}^{\infty} dk_1^0 \int \prod_{i=1}^{3}\frac{d^3k_i}{(2\pi)^3}\frac{1}{2E_2 2E_3} S_D(k_1^0,\mathbf{k}_1)$$
$$\times (2\pi)^4\delta^{(3)}(\mathbf{k}+\mathbf{k}_3-\mathbf{k}_1-\mathbf{k}_2)\delta(k^0+\lambda' E_3-\lambda E_2-k_1^0)$$
$$\times |T(k^0+\lambda' E_3,\mathbf{k}+\mathbf{k}_3)|^2 f^{(0)}(\lambda' E_3)\tilde{f}^{(0)}(\lambda E_2)\tilde{f}^{(0)}(k_1^0)$$

- Thermal effects in $|T|^2$ and $E_k$
- Landau cut
- Off-shell effects

# RESULTS: $D$ MESON TRANSPORT COEFFIECIENTS

[J. Torres-Rincon, GM, A. Ramos, L. Tolos (in preparation)]

In the static limit $\mathbf{k} \to 0$

For $k^0 = E_k$ solution of

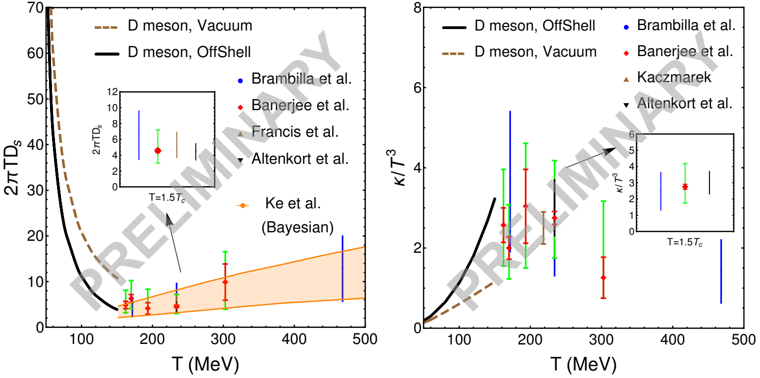
$$E_k^2 - \mathbf{k}^2 - m_D^2 - \mathrm{Re}\,\Pi(E_k, \vec{k}; T) = 0$$

Spatial diffusion coefficient

$$2\pi T D_s(T) = \lim_{\mathbf{k}\to 0} \frac{2\pi T^3}{\hat{B}_0(E_k, \mathbf{k}; T)}$$

Momentum diffusion coefficient

$$\kappa(T) = 2\hat{B}_0(E_k, \mathbf{k} \to 0; T)$$




Good matching around $T_c$ of our results with the lattice QCD data and a Bayesian analysis, specially when thermal and off-shell effects are included.

# Conclusions

# CONCLUSIONS

- We have described the scattering of open charm mesons off light mesons including temperature corrections in a self-consistent manner.
- We have obtained spectral functions at various temperatures below $T_c$.
- The mass of the open-charm ground-state mesons decreases with temperature while they acquire a substantial width.
- Modification also of the dynamically generated resonances, but still far from chiral degeneracy at the temperatures explored.
- The largest effect comes form the pions in the bath. Heavier light mesons are less abundant.
- We have obtained Euclidean correlators from spectral functions at unphysical masses, which are in good agreement with LQCD results well below $T_c$. The the discrepancy close to $T_c$ indicates the missing contribution of higher-excited states.
- We have introduced thermal and off-shell effects in the computattion of $D$-meson transport coefficients. The Landau Cut contributes sizeably at moderate temperatures.