Theoretical approach to correlation functions of strange hadrons at accelerator experiments and search for exotic bound states

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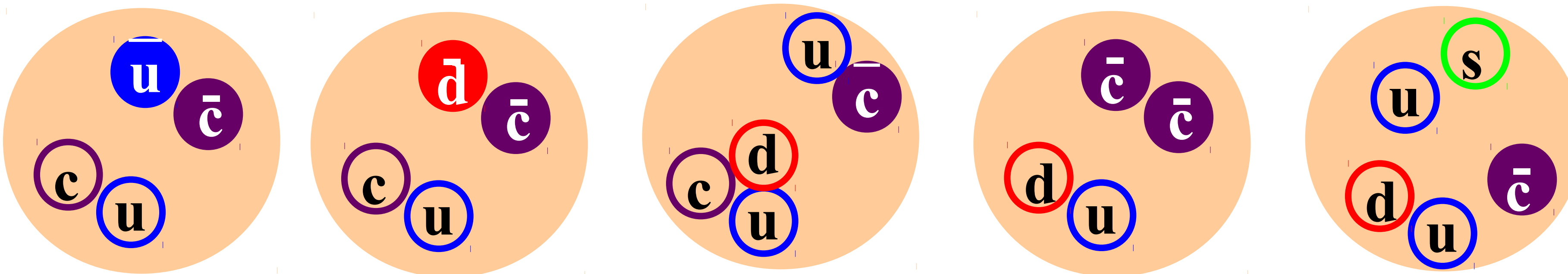
- **Introduction**
  - **Femtoscopic study of hadron-hadron interaction: Basics**
- **Bound state diagnosis from femtoscopy**
  - **Do we have a bound state in NΩ, N$\overline{\mathrm{K}}$, and NΞ ?**
- **Other hadron-hadron correlation functions**
- **Summary**

Introduction
– Femtoscopic study of hh interactions –

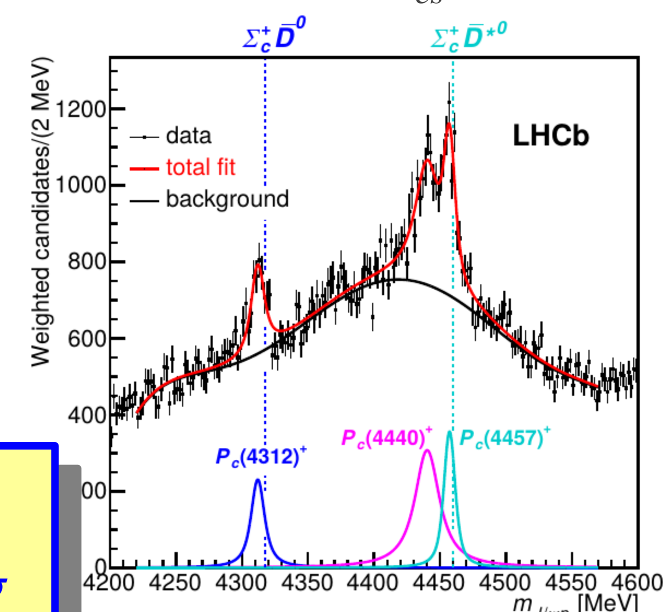
Exotic Hadrons

- **Exotic hadrons**
  → X, Y, Z, Pc .... Discovered/Proposed
  at LEPS, Belle, BaBar, CLEO, BES(I,II,III), LHCb, ...



X(3872) **Z(4430)** **Pc(4450)** $\mathbf{T_{cc}}$ Θ^{+}_{cs}

- **Proposed mechanisms: Singularity, Hadronic molecule, Compact multi-q state, Hadro-quarkonium, glueball, ...**



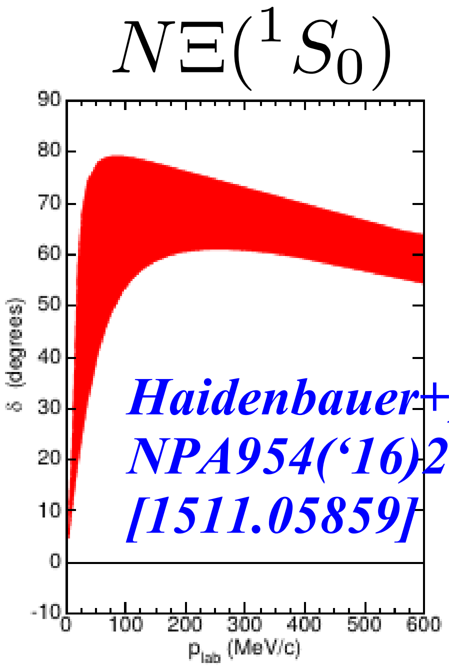
LHCb [1904.03947]

Various hadron-hadron (hh) interactions need to be known for deeper understanding

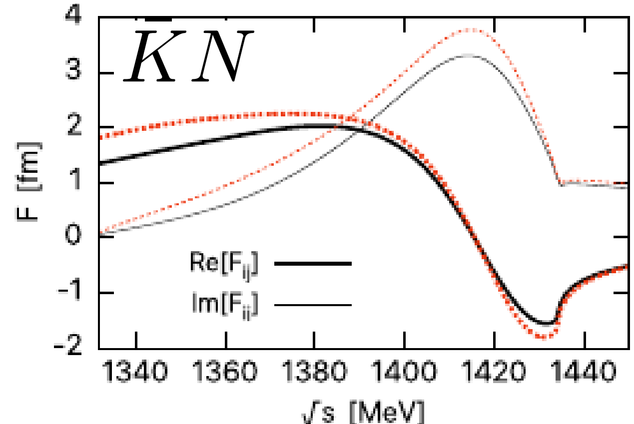
How can we access hh interactions ?

- Theoretical approaches
  - Nuclear force models: meson exch., quark model, … (need data)
  - *Ab initio*: chiral EFT (χEFT), lattice QCD (need data or CPU resources)

Chiral

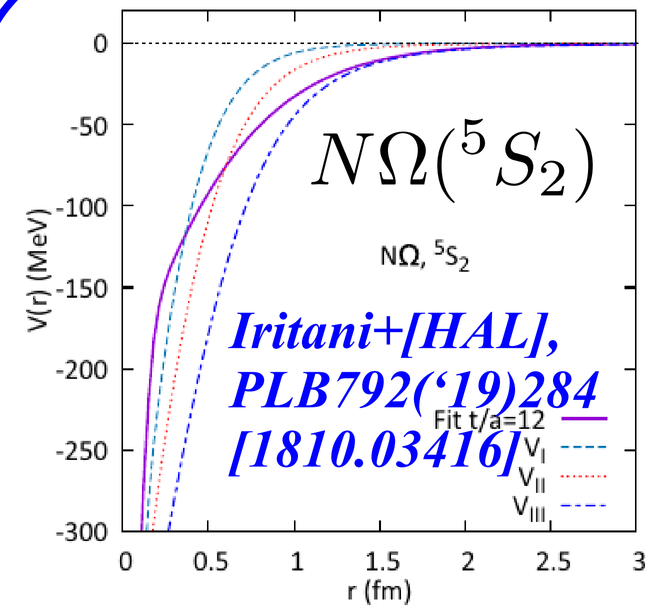


Haidenbauer+, NPA954('16)273 [1511.05859]

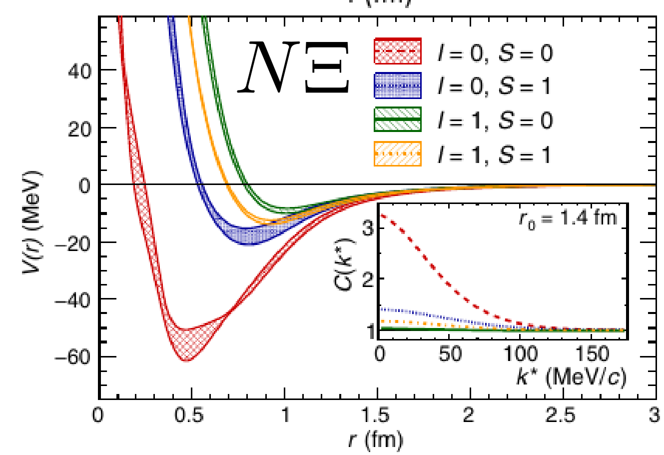


Miyahara, Hyodo, Weise, PRC98('18), 025201 [1804.08269] (Ikeda-Hyodo-Weise amplitude)

Lattice



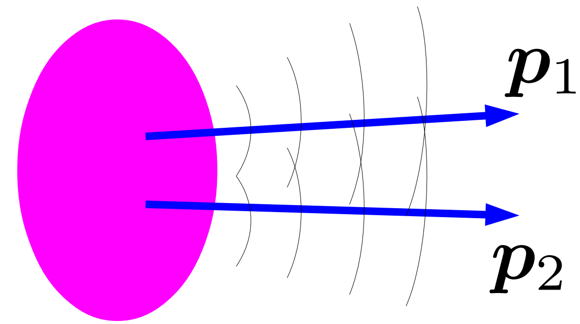
Iritani+[HAL], PLB792('19)284 [1810.03416]



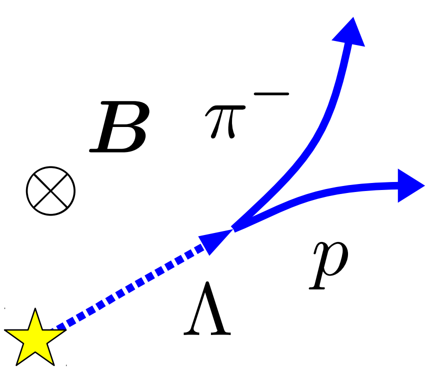
Sasaki+ [HAL], NPA998 ('20)121737 [1912.08630] (taken from ALICE('19))

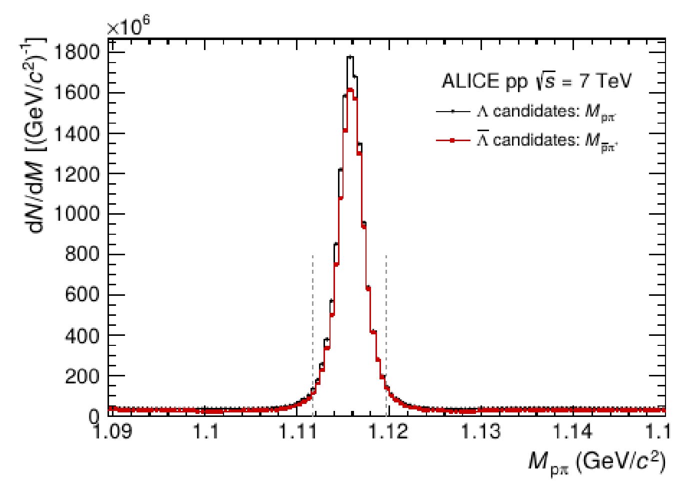
How can we access hh interactions ?

- **Experimental approaches**
  - **hh scattering (NN, YN, πN, KN)**
  - **Hadronic nuclei (normal nuclei, hypernuclei, kaonic nuclei)**
  - **Hadronic atom (π^-, K^-, Σ^-, Ξ^-, …)**
  - **Femtoscopy**
- **Femtoscopic study of hh interactions**
  - **Applicable to various hh pairs (NN, YN, KN, DN, YY, Yd, YNN, …)**
  - **Valid when the source is chaotic**
  - **Weakly decaying particles → Good pair purity**
  - **Future measurements: Charmed hadron, hNN, ...**



$$C(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1) N_2(\boldsymbol{p}_2)}$$

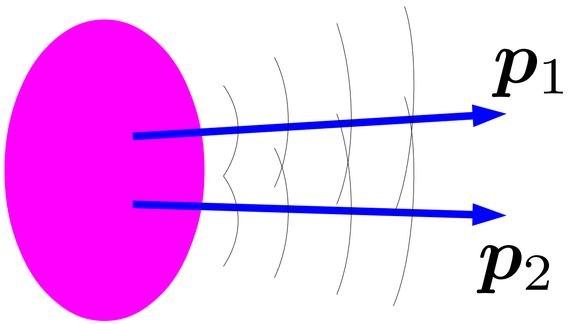




ALICE [1805.12455]

Femtoscopic study of hadron-hadron interaction

- **Correlation function (CF)**
  - **CF = convolution of source fn. and |w.f.|^2 (Koonin-Pratt formula)**
    Koonin('77), Pratt+('86), Lednicky+('82)



$$C(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{N_{12}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{N_1(\boldsymbol{p}_1) N_2(\boldsymbol{p}_2)} \simeq \int d\boldsymbol{r}\, \underbrace{S(\boldsymbol{r})}_{\text{source fn.}} \underbrace{|\varphi_{\boldsymbol{q}}(\boldsymbol{r})|^2}_{\text{relative w.f.}}$$

- **Source size from quantum stat. + CF**
  Hanbury Brown & Twiss ('56); Goldhaber, Goldhaber, Lee, Pais ('60)
- ***Hadron-hadron interaction from source size + CF***
  - **CF of non-identical pair from static spherical source**
    R. Lednicky, V. L. Lyuboshits ('82); K. Morita, T. Furumoto, AO (1408.6682)

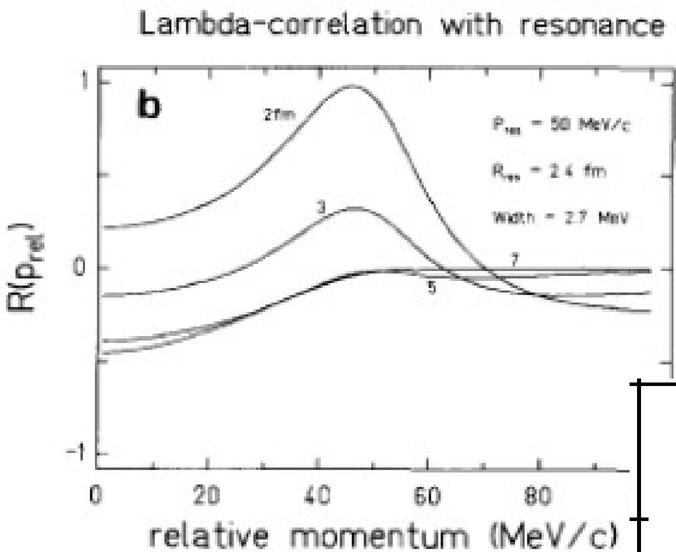
$$C(\boldsymbol{q}) = 1 + \int d\boldsymbol{r}\, S(r) \left\{ |\varphi_0(r)|^2 - |j_0(qr)|^2 \right\} \quad (\varphi_0 = \text{s-wave w.f.})$$

CF shows how much $|\varphi|^2$ is enhanced → V_{hh} effects !

Example: ΛΛ correlation and ΛΛ interaction

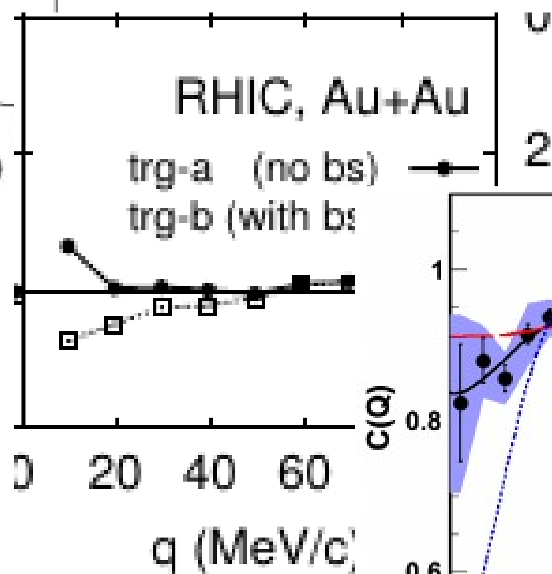
$$C(\boldsymbol{q}) = 1 - \frac{\lambda}{2} e^{-4q^2R^2} + \frac{\lambda}{2}\int d\boldsymbol{r} S(r)\left\{|\varphi_0(r)|^2 - |j_0(qr)|^2\right\}$$

λ = pair purity prob.

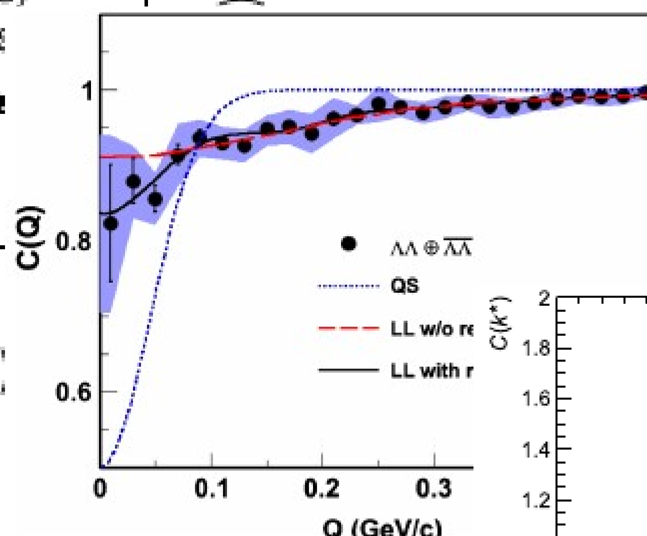


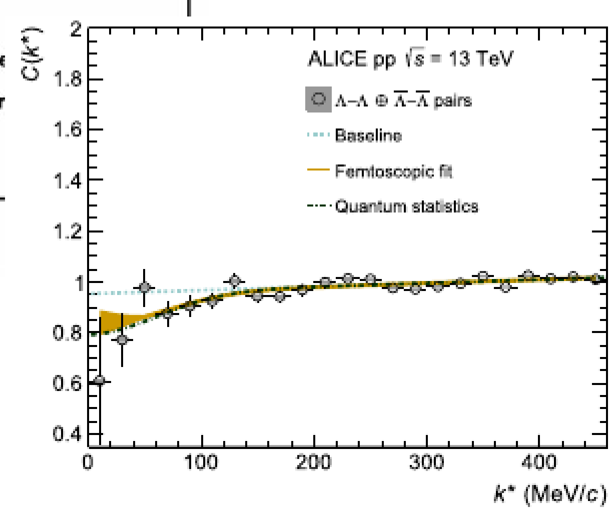
C. Greiner, B. Muller, PLB219('89)199.

AO, Hirata, Nara, Shinmura, Akaishi, NPA670('00)297c



L. Adamczyk+[STAR], PRL114('15)022301

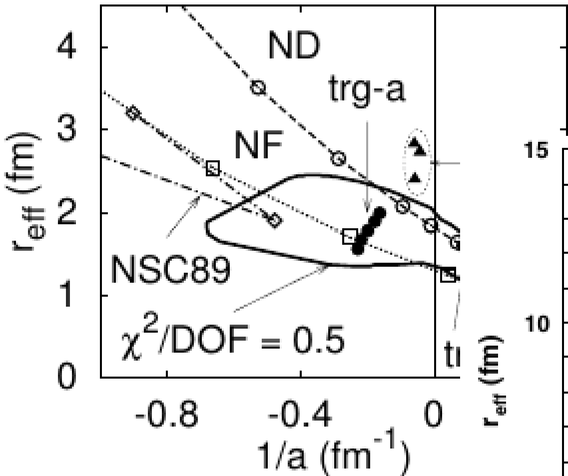




Slight enh. over quantum statistical (HBT) CF.

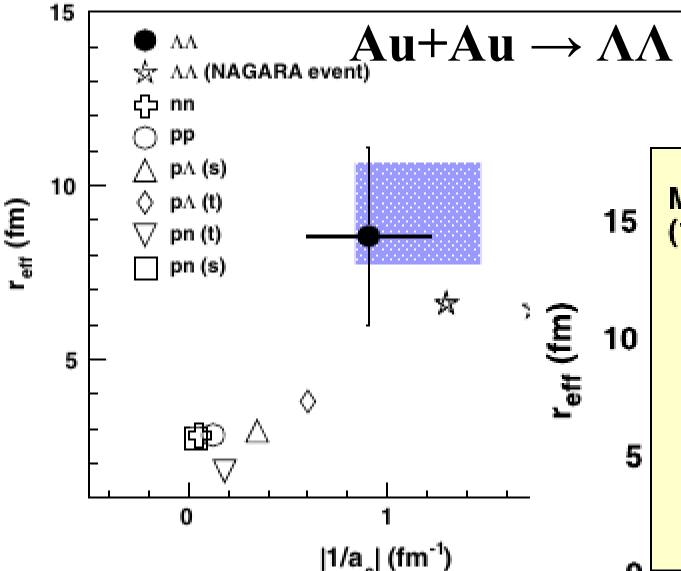
S. Acharya+[ALICE], PLB797('19)134822

Example: ΛΛ correlation and ΛΛ interaction

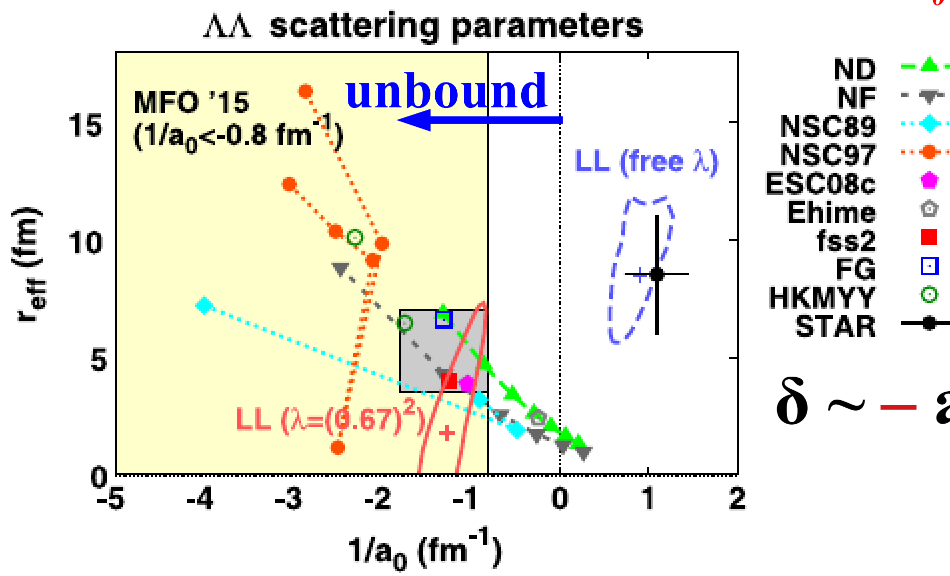


AO+('00)
(Before Nagara,
from (K^-, $K^+\Lambda\Lambda$)
inv. mass spec.)

L. Adamczyk+[STAR],
PRL114('15)022301



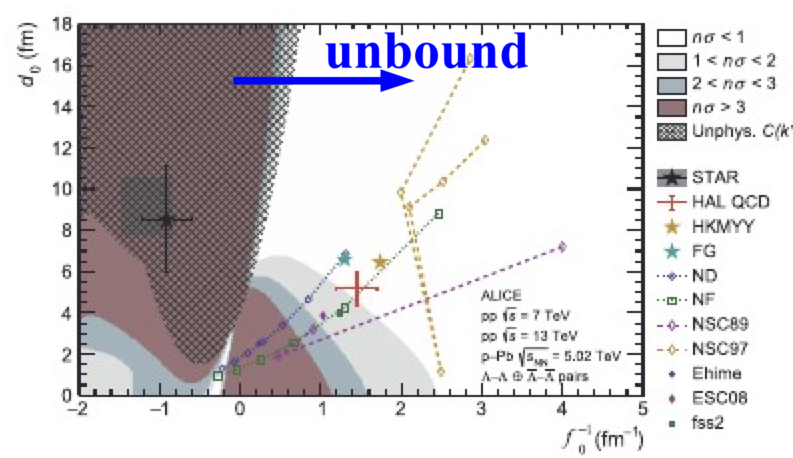
AO, K. Morita, K. Miyahara,
T. Hyodo, NPA954('16)294;
Morita, Furumoto, AO, PRC
91((15)024916. -1.25 fm < a_0 < 0.



$\delta \sim - a_0 q$

S. Acharya+[ALICE],
PLB797('19)134822

$\delta \sim + a_0 q$



It is unlikely that ΛΛ bound state exists.

Which hh interactions are accessible ?

Scatt.+Nuclei

Scatt.+Mesic atom

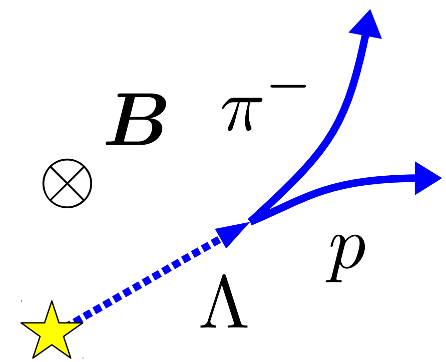
Scatt. +Hyper Nuclei

	n	p	K^-	K^+	π^-	π^+	Λ	Σ	Ξ^-	Ω^-
n										
p		O	O	O	△	△	O	O	O	O
K^-		O	O	O	O	O				
K^+		O	O	O	O	O				
π^-		△	O	O	O	O				
π^+		△	O	O	O	O				
Λ		O					O			
Σ		O								
Ξ^-		O								
Ω^-		O								

Current Femtoscopy (O: observed and analyzed)

ΛΛ hypernuclei

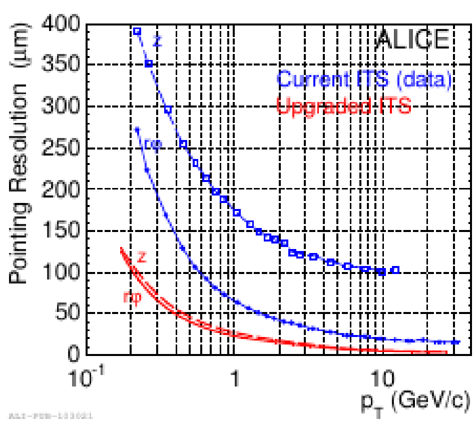
How far do hadrons fly ?

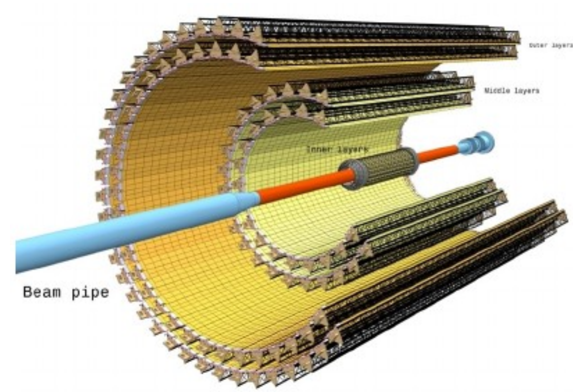


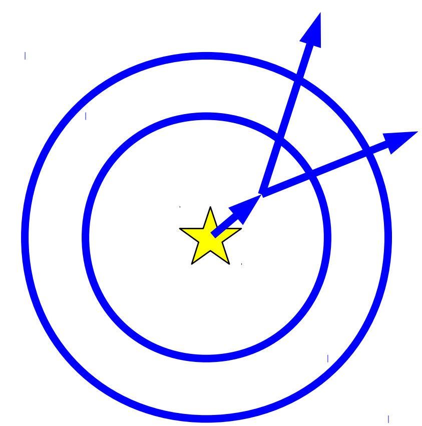
- **Average flight length**

$$\ell = \gamma v \tau = \gamma \beta c \tau \simeq \gamma (c\tau)\ (E \gg m)$$

- **Hadron cτ**
  - **Strange baryons → A few cm → Time Projection Chamber (TPC)**
    - **$c\tau(\Lambda)$=7.9 cm, $c\tau(\Sigma^+)$=2.4 cm, $c\tau(\Xi^-)$=4.9 cm.**
  - **Charmed hadrons → A few hundred μm → Silicon Vertex Detector**
    - **$c\tau(D^\pm)$=312 μm, $c\tau(D^0)$=123 μm, $c\tau(\Lambda^+)$=61 μm,**







It will be possible to measure each charmed hadron !

G. Contin+[ALICE] PoS(Vertex2019)003

Which hh interactions are accessible ?

Scatt.+Nuclei

Scatt.+Mesic atom

Scatt.
+Hyper
Nuclei

	n	p	K⁻	K⁺	π^-	π^+	Λ	Σ	Ξ^-	Ω^-	D^-	D^+	Ks	+α
n														
p		O	O	O	△	△	O	O	O	O	O	O		
K^-		O	O	O	O	O								
K^+		O	O	O	O	O								
π^-		△	O	O	O	O								
π^+		△	O	O	O	O								
Λ		O					O							
Σ		O												
Ξ^-		O												
Ω^-		O												
D^-		O												
D^+		O												
Ks														
+α														

Current
Femtoscopy
(O: observed
and analyzed)

ΛΛ hypernuclei

Future Femtoscopy
(High-luminosity
+ Silicon Vertex Detector)

Factor 3 more hh int. will be accessible in the near future.

Bound state diagnosis using femtoscopy

Bound state vs Source size dependence of CF

- **To be bound, or not to be bound ?**
  - **Leading 6q dibaryon candidates: H (ΛΛ-NΞ-ΣΣ), NΩ, ΔΔ(=d*) (No Pauli blocking of quarks, Color-spin int. is attractive)**
    A. Gal ('16); M. Oka ('88)
  - **Mesons: σ (ππ), f_0/a_0 ($K\bar{K}$), X(3872)($D\bar{D}^*$)**
  - **Pentaquark state: Pc(4450) ($\Sigma_c\bar{D}^*$)**
- **Does CF depends on the existence of a bound state ? → Yes**
  - **Lednicky-Lyuboshits analytic model shows specific size dependence (Asymp. w.f.+eff. range corr.+Gaussian source)**
    Lednicky, Lyuboshits ('82)

$$\psi_0(r) \to \psi_{\rm asy}(r) = \frac{e^{-i\delta}}{qr}\sin(qr+\delta) = \mathcal{S}^{-1}\left[\frac{\sin qr}{qr} + f(q)\frac{e^{iqr}}{r}\right]$$

$$\boxed{\begin{aligned} C_{\rm LL}(q) =& 1 + \int d\boldsymbol{r} S_{12}(r)\left(|\psi_{\rm asy}(r)|^2 - |j_0(qr)|^2\right) \\ =& 1 + \frac{|f(q)|^2}{2R^2}F_3\left(\frac{r_{\rm eff}}{R}\right) + \frac{2{\rm Re}f(q)}{\sqrt{\pi}R}F_1(2x) - \frac{{\rm Im}f(q)}{R}F_2(2x)\end{aligned}}$$

$(x = qR, R = \text{Gaussian size}, F_1, F_2, F_3 : \text{Known functions})$

Source Size Dependence of Correlation Function

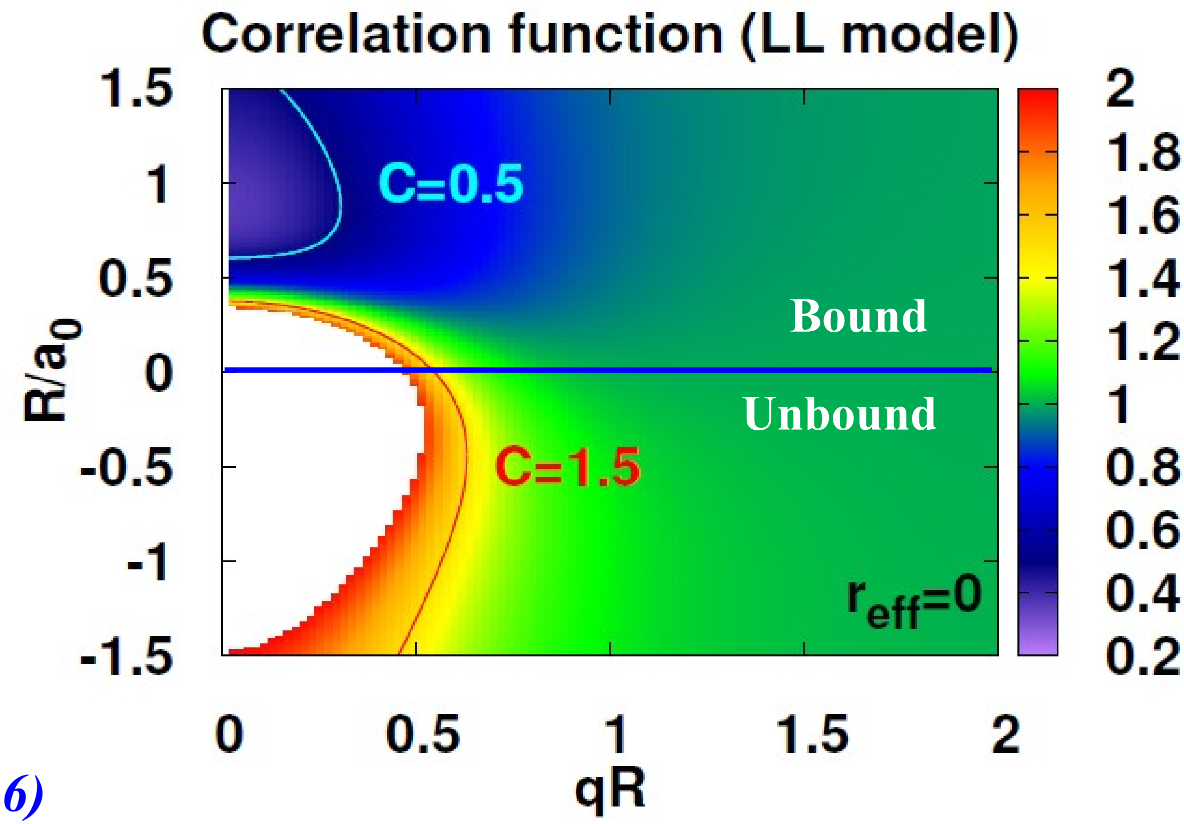
- "Zero-range" case in LL model

$$r_{\rm eff} = 0 \to q\cot\delta = -1/a_0 \to f(q) = (q\cot\delta - iq)^{-1} = -\frac{R}{R/a_0 + iqR}$$

$$C(x,y) = 1 + \frac{1}{x^2+y^2}\left[\frac{1}{2} - \frac{2y}{\sqrt{\pi}}F_1(2x) - xF_2(2x)\right] \quad (x = qR, y = R/a_0)$$

$$= \frac{1}{2}\left(\frac{1}{y} - \frac{2}{\sqrt{\pi}}\right)^2 + 1 - \frac{2}{\pi} \quad (F_1 \to 1, F_2 \to 0 \text{ at } x \to 0)$$

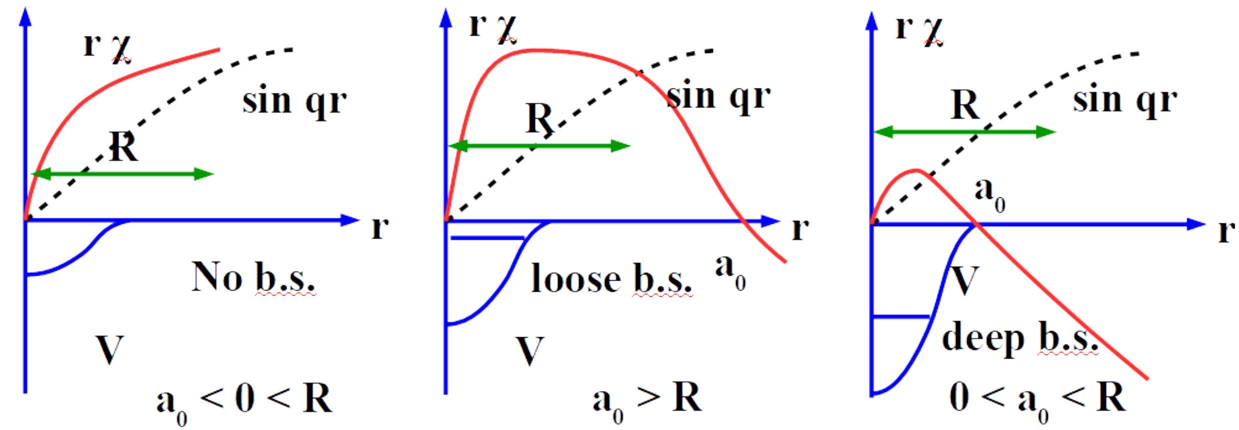
  - **C(q) takes a minimum value 1-2/π ~0.36 at $R/a_0=\sqrt{\pi}/2\sim0.89$**

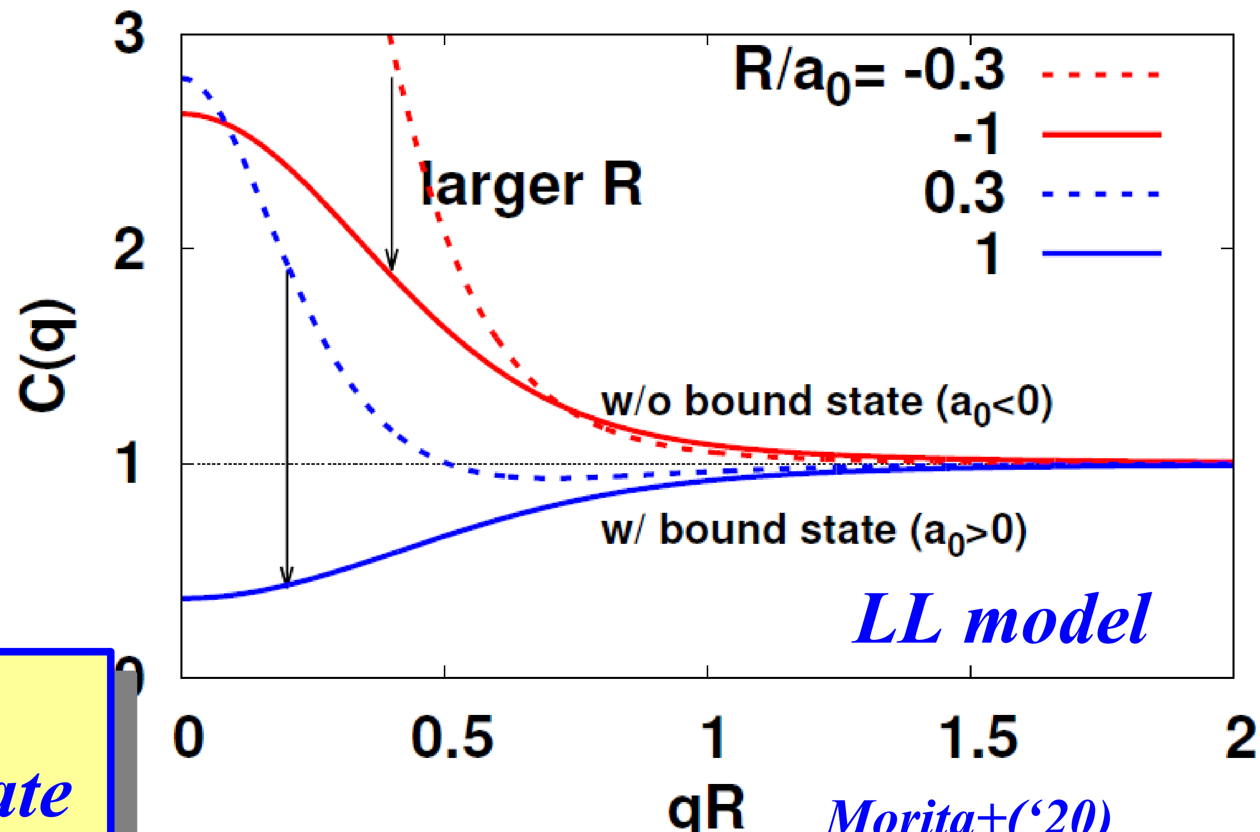


E.g. AO, Morita, Miyahara, Hyodo ('16)

From correlation function to hadron-hadron interaction

- **With a bound state and for $R \sim a_0$ $(a_0>0)$, $|\mathrm{w.f.}|^2$ is suppressed in the source region → Suppressed C(q)**



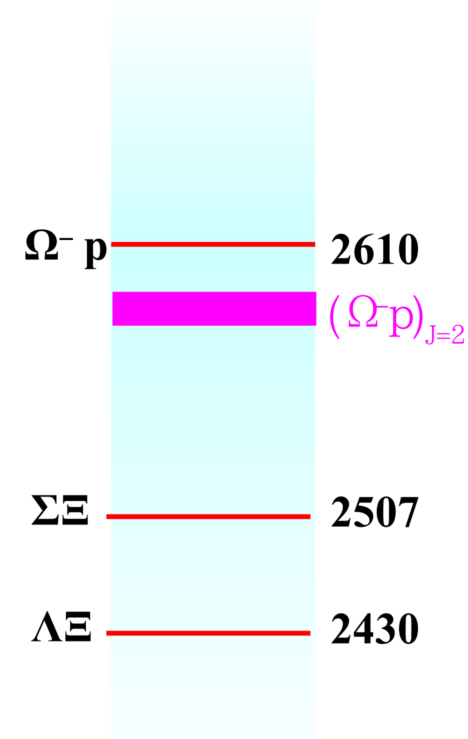


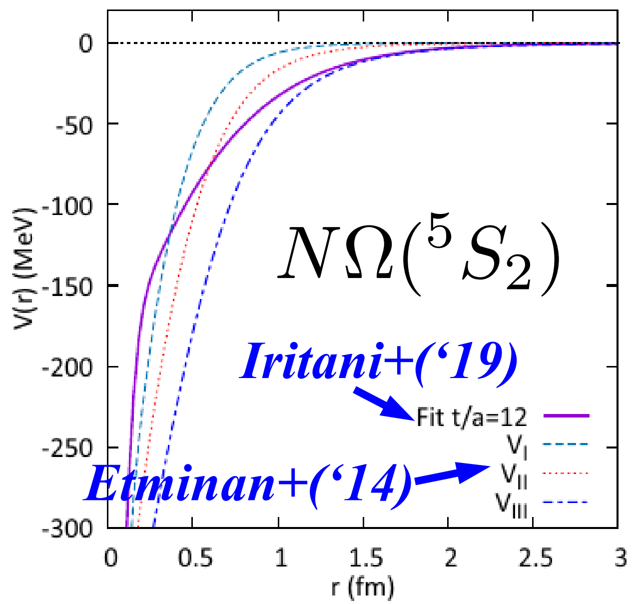
Morita+('20)

Source size dep. of CF → Existence of bound state

Example 1: NΩ interaction and NΩ bound state

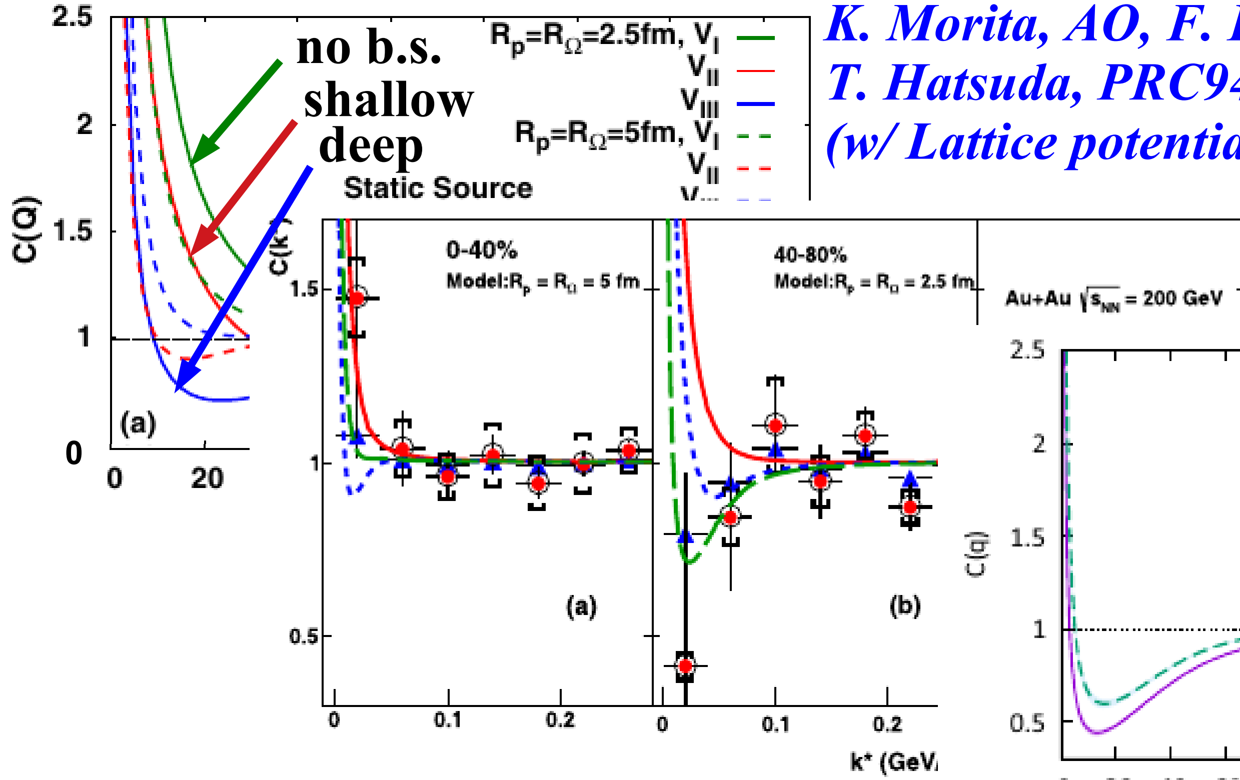
- Ω^- : quark content=sss, J^π=3/2+, M=1672 MeV
- Ω^- p bound state as a S= –3 dibaryon ?
  - No quark Pauli blocking
    in ΩN, H=uuddss, and d*=ΔΔ channels.
    Oka ('88), Gal ('16)
  - J=2 state ($^5S_2$) couples to Octet-Octet
    baryon pair only with L ≥ 2
    → Small width is expected.
    T. Goldman+, PRL59('87),627;
    F. Etminan+[HAL], NPA928('14)89;
    Iritani+[HAL], PLB792('19)284.
  - Correlation has been measured
    at RHIC & LHC !
    STAR ('19); ALICE ('20)





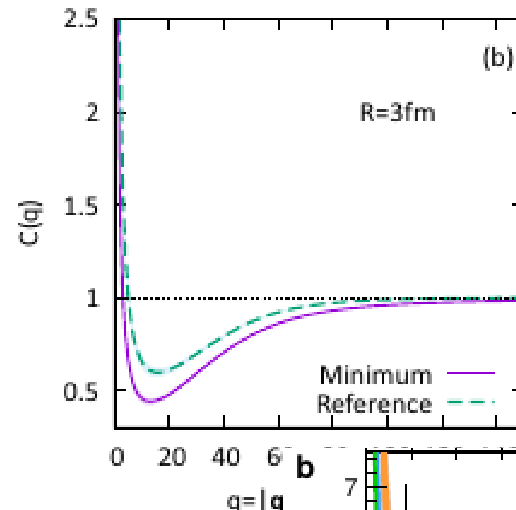
Let us try to discover the first S<0 dibaryon !

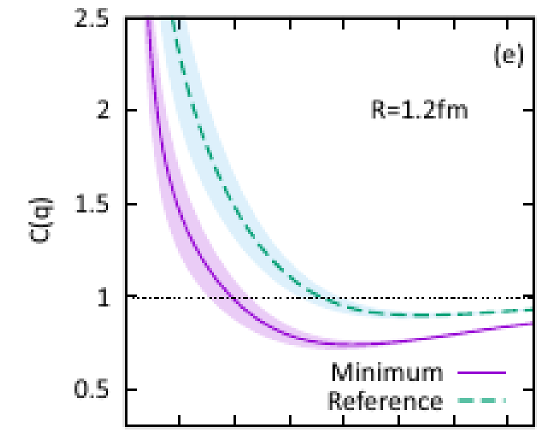
$p\Omega^-$ correlation function



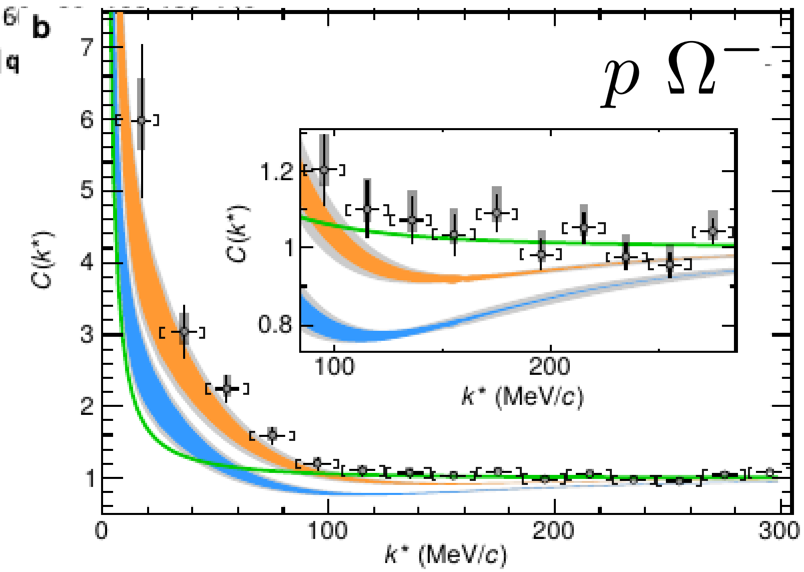
K. Morita, AO, F. Etminan, T. Hatsuda, PRC94('16)031901(R) (w/ Lattice potential with heavier quark mass)

J. Adam+[STAR], PLB790('19)490.

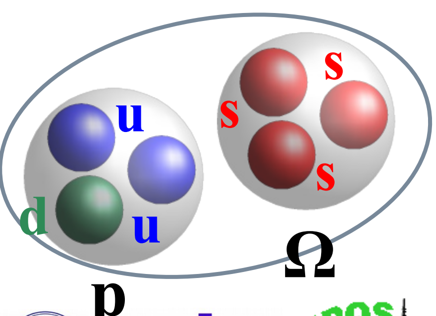




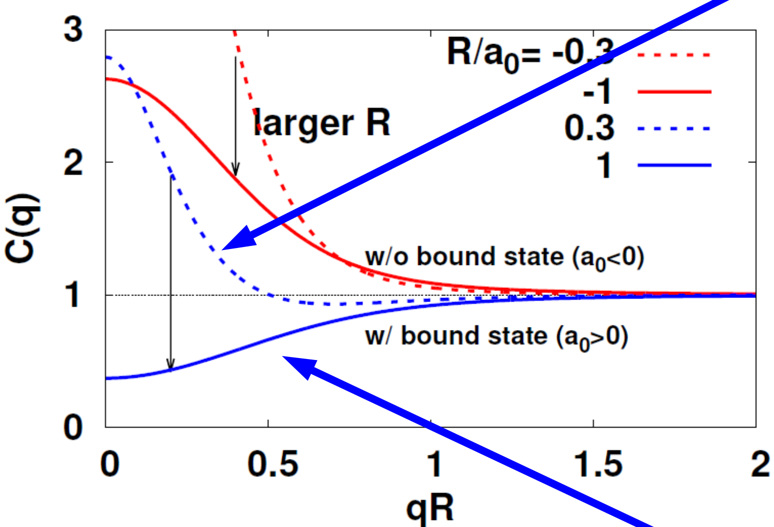
K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC 101('20)015201. (w/ Lattice potential at physical quark mass, $a_0 \sim 3.4$ fm)

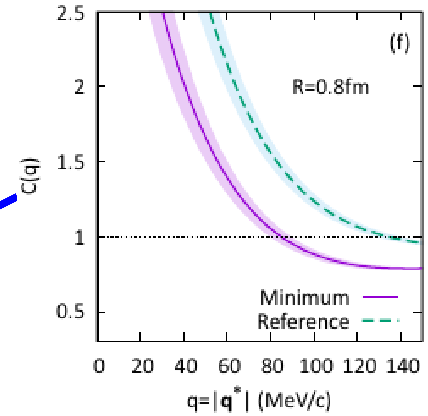


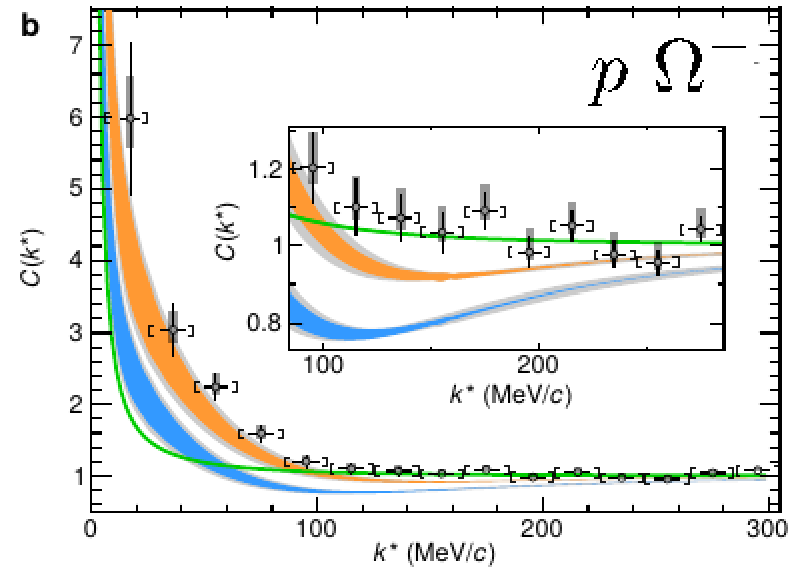
S. Acharya+[ALICE], Nature 588 ('20), 232 [2005.11495] (pp 13 TeV)



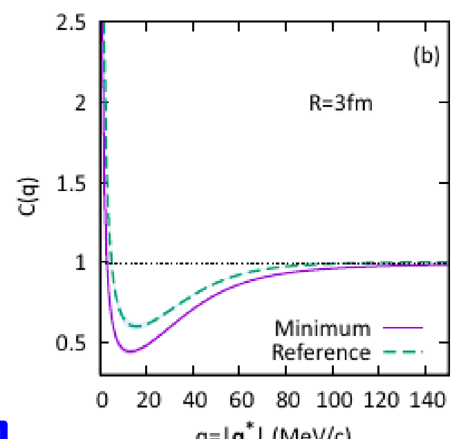
STAR + ALICE = NΩ Dibaryon

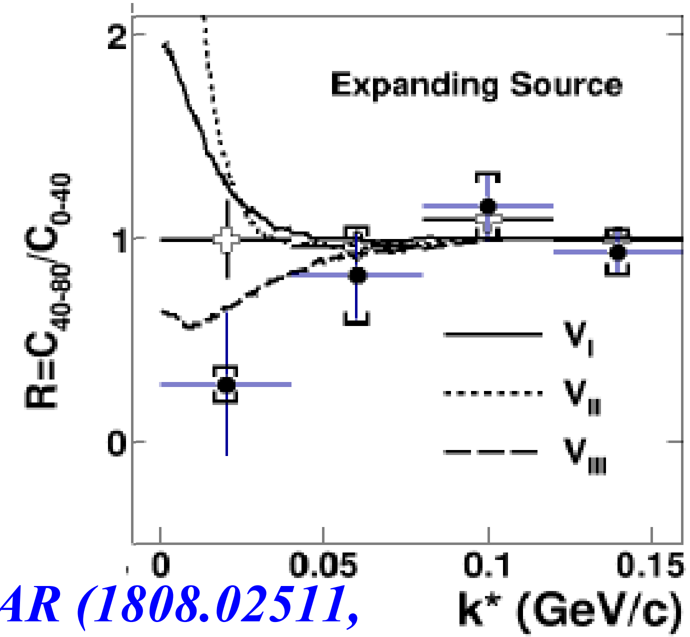






ALICE, Nature588('20)232 [2005.11495]



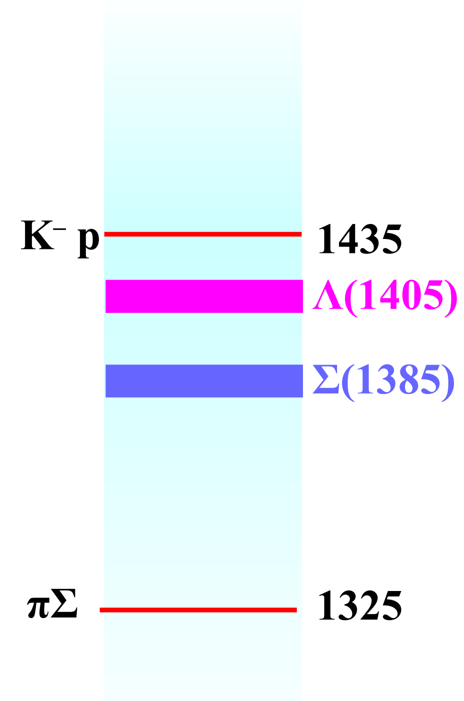


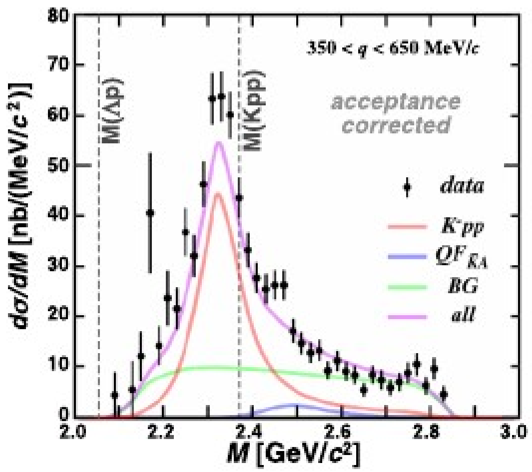
STAR (1808.02511, PLB790 ('19) 490)

Dip from a bound state survives Coulomb.

Example 2: N $\bar{K}$ interaction and Λ(1405)

- **Λ(1405) $\bar{K}$N quasi-bound state**
  Dalitz, Tuan ('60); Koch ('94); Kaiser, Siegel, Weise ('95); AO, Nara, Koch ('97)
  - **Positive scattering length in K^- atoms (c.f. Del Grande, Zmeskal)**
    M.Iwasaki et al. PRL78('97)3067;
    M.Bazzi et al. [SIDDHARTA Collab.], PLB704('11)113.
- **Kaonic nuclei ?**
  Nogami ('63); Akaishi, Yamazaki ('02); Shevchenko, Gal, Mares ('07); Ikeda, Sato ('07); Dote, Hyodo, Weise ('09);
  S.Ajimura+ [J-PARC E15], PLB 789 (2019) 620.
  → Needs precise info. on $\bar{K}$N int.
- **Scattering amplitude and Potential fitting scattering and SIDDARTA data in chiral approach**
  Ikeda, Hyodo, Weise ('11,'12);
  A. Cieplý, J. Smejkal ('12, NLO30);
  Miyahara, Hyodo, Weise ('18, CC N$\bar{K}$-πΣ-πΛ potential)



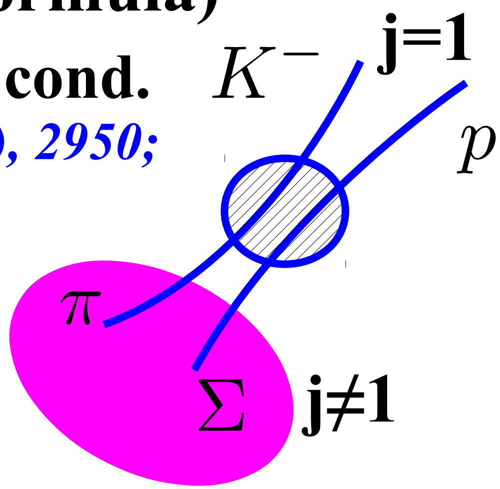


J-PARC E15 ('19)

How about K^- p correlation ?

Correlation Function with Coupled-Channel Effects

- **To evaluate pK⁻ correlation function, we need to take account of coupled-channel effects of NK-πΣ !**
- **Correlation function formula with CC (KPLLL formula)**
  - **Coupled-channel contributions with ψ⁽⁻⁾ boundary cond.**
    Lednicky, Lyuboshits, Lyuboshits, Phys. Atom. Nucl. 61 (1998), 2950;
    J. Haudenbauer, NPA981('19)1 [1808.05049].



$$C(\boldsymbol{q}) = \int d\boldsymbol{r} \sum_j \omega_j S_j(\boldsymbol{r}) |\Psi_j^{(-)}(\boldsymbol{r})|^2$$

$$= 1 - \int d\boldsymbol{r} S_1(r) |j_0(qr)|^2 + \int d\boldsymbol{r} \sum_j \omega_j S_j(r) |\psi_j^{(-)}(q;r)|^2$$

$$\psi_{j=1}(r) \to [e^{iqr} + A_1(q) e^{-iqr}]/2iqr \quad (\omega_1 = 1)$$

$$\psi_{j\neq 1}(r) \to A_j(q) e^{-iqr}/2iqr \quad [\Psi^{(-)} \text{ boundary condition}]$$

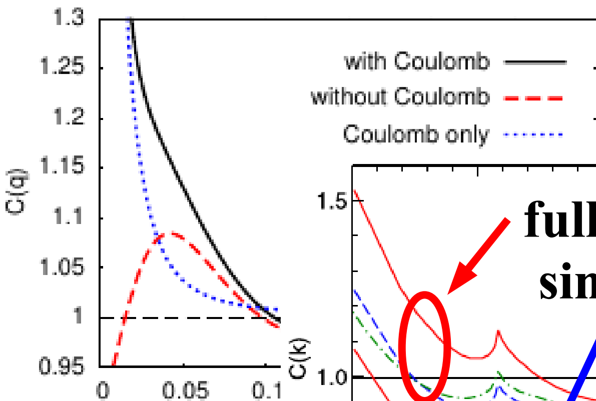
$$\underbrace{\omega_j}_{\text{Source weight}} \underbrace{S_j(\boldsymbol{r})}_{\text{Normalized Source fn.}} |\psi_j^{(-)}(q;r)|^2$$

Source weight **Normalized Source fn.**

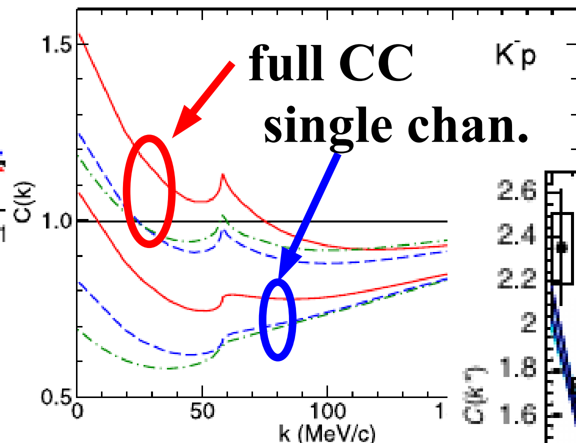
(No Coulomb case)

  - **Effects of coupled-channel, strong & Coulomb pot., and threshold difference are taken into account in the charge base, pΞ⁻, nΞ⁰, ΛΛ.**
    Y. Kamiya+, PRL('20, K⁻ p)
  - **Source size R and weight ω_j (j≠1) are taken as the parameter.**

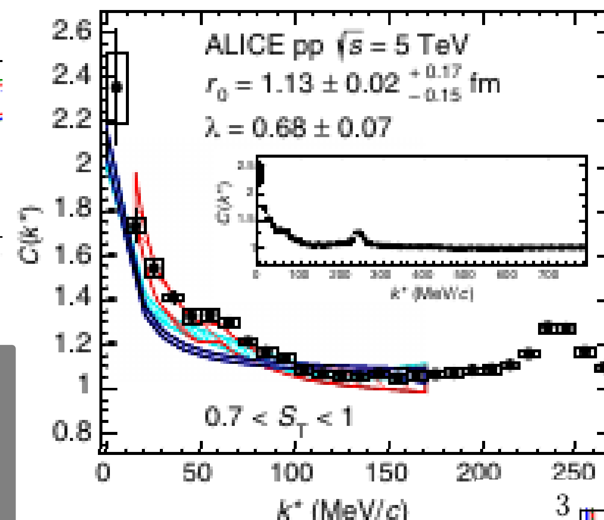
pK^- correlation



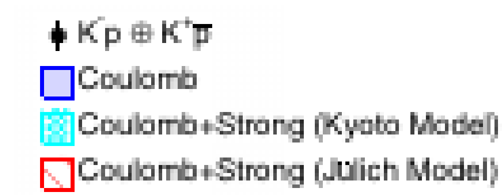
S. Cho+ [ExHIC], PPNP95('17)279.
(Insufficient coupled-channel effects)



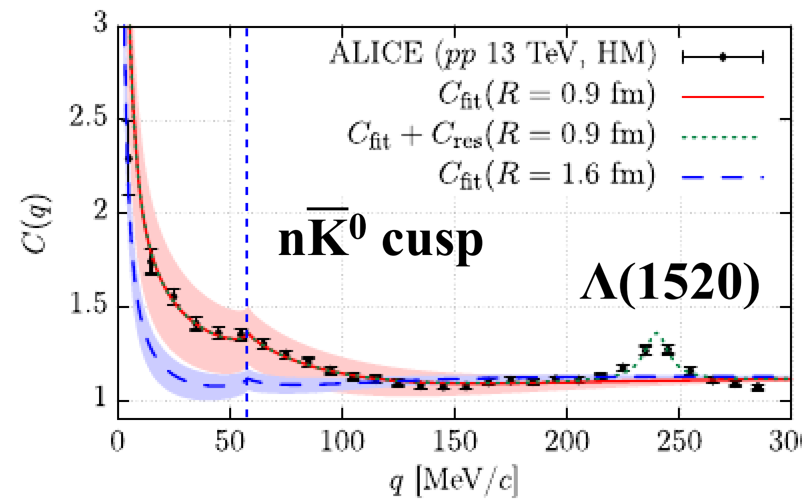
J. Haidenbauer, NPA981('19)1.
Julich, NLO30, w/ CC effects,
w/o Coulomb)



S. Acharya+[ALICE], PRL124('20)092301



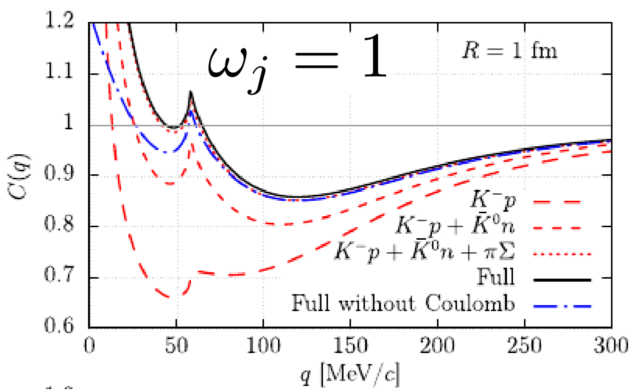
CF with small source is explained ! Source size dep. may clarify bound state nature of Λ(1405).

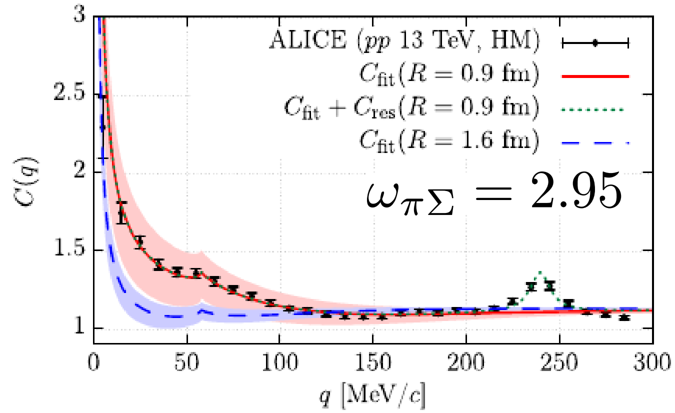


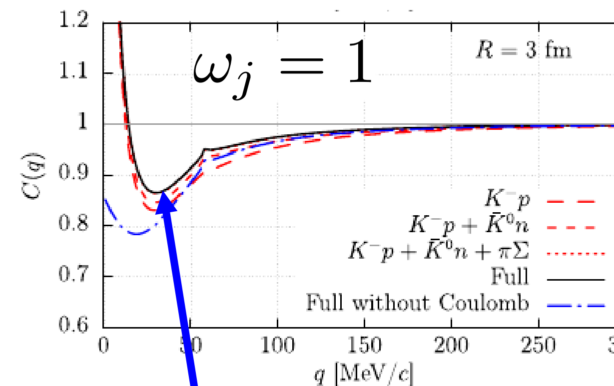
Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.
(Chiral SU(3) dynamics)

Source Size Dependence of C(pK⁻)

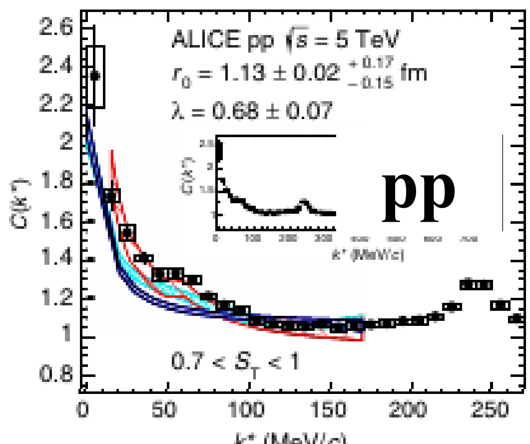
- **Coupled-channel effects are suppressed when R is large, and "pure" pK⁻ wave function may be observed in HIC.**

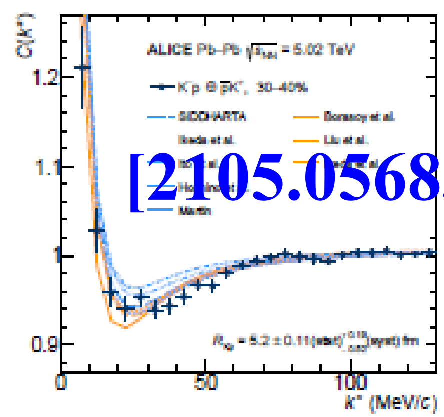






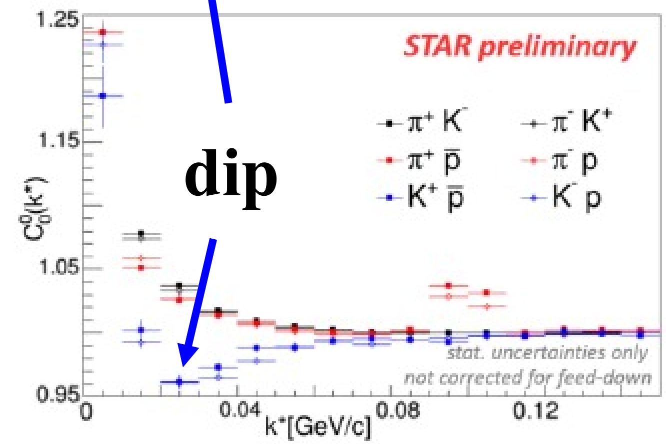
Y. Kamiya, T. Hyodo, K. Morita, AO, W. Weise, PRL124('20)132501.





[2105.05683]

dip



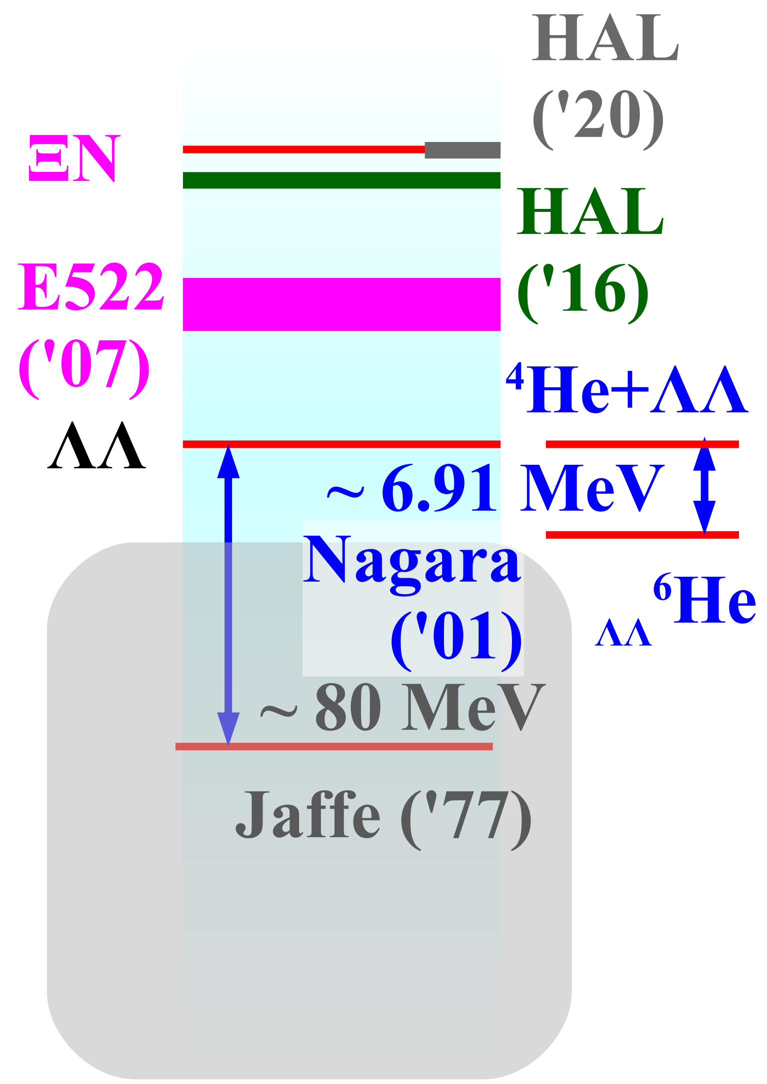
S. Acharya+[ALICE], PRL124('20)092301 *Siejka+[STAR, preliminary], NPA982 ('19)359.*

STAR preliminary / new ALICE data seems to show a dip, which suggests the existence of a bound state.

Do I have 5 minutes ?

H dibaryon state, to be bound or not to be bound ?

- **H-dibaryon: 6-quark state (uuddss)**
  - **Prediction:** *R.L.Jaffe, PRL38(1977)195*
  - **Ruled-out by double Λ hypernucleus**
    Takahashi et al.,PRL87('01) 212502
  - **Resonance or Bound "H" ?**
    Yoon et al.(KEK-E522)+AO ('07)
- **Lattice QCD results**
  - **Bound (below ΛΛ threshold):**
    HALQCD('11), NPLQCD('11,'13), Mainz('19)
    (heavier quark mass or SU(3) limit)
  - **Resonance (Bound state of NΞ):**
    HAL QCD ('16,18) **(HAL preliminary)**
  - **Virtual Pole (around NΞ threshold)**
    HAL QCD ('20) **(almost physical m_q)**



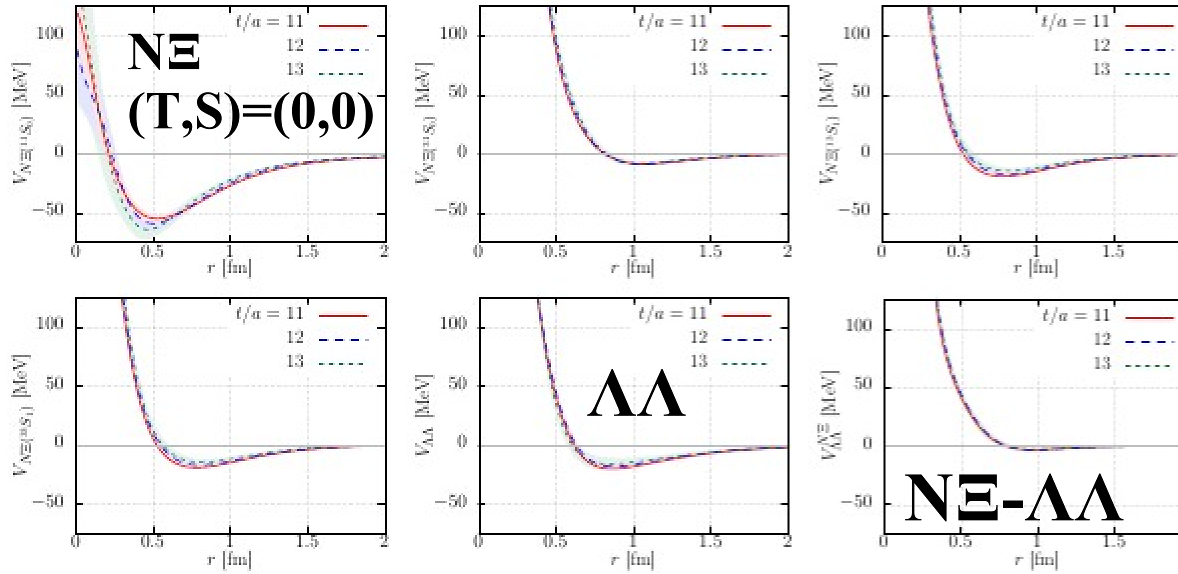
We examine LQCD NΞ-ΛΛ potential and discuss H using CF !

NΞ-ΛΛ potential from Lattice QCD

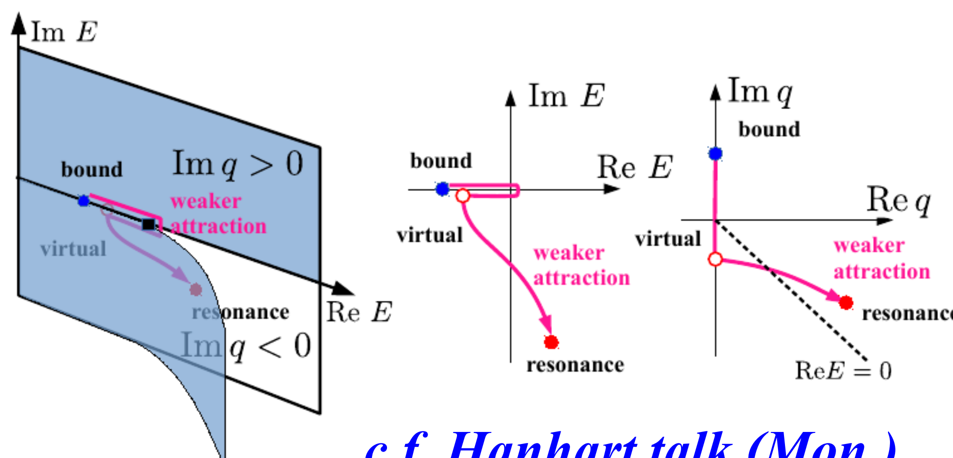
- **NΞ-ΛΛ potential at almost physical quark mass (m_π=146 MeV) by HAL QCD Collaboration**

  K. Sasaki et al. [HAL QCD Collab.], NPA 998 ('20) 121737 (1912.08630)

  - **Strong attraction in (T,S)=(0,0) of NΞ**
  - **Weak attraction in ΛΛ (Coupling with NΞ causes ΛΛ attraction)**
  - **There is no bound state in NΞ-ΛΛ system (except for Ξ^- atom), but there is a virtual pole around the NΞ threshold (3.93 MeV below $n\Xi^0$ threshold) on the irrelevant Riemann sheet, (+, –, +) [relevant=(–,+,+)]**

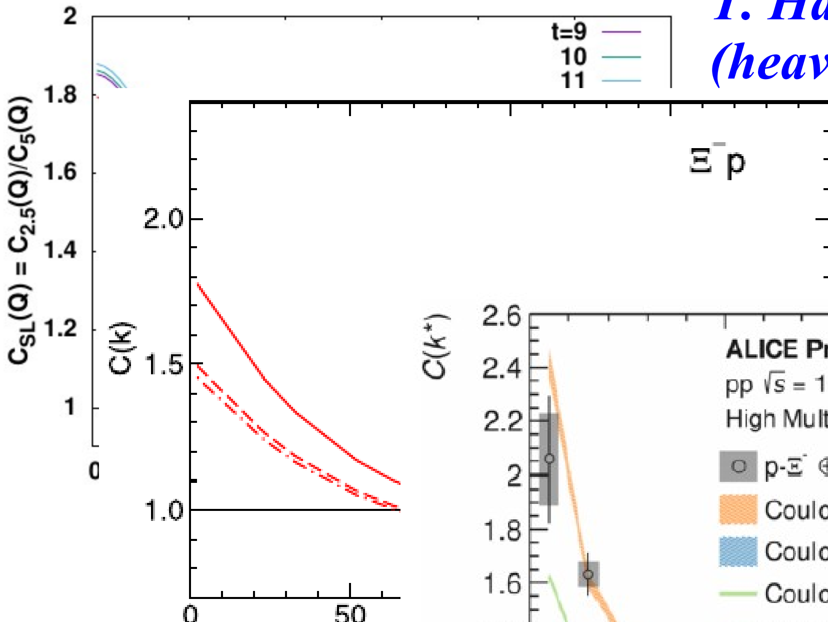


sign of Im(eignen momentum)



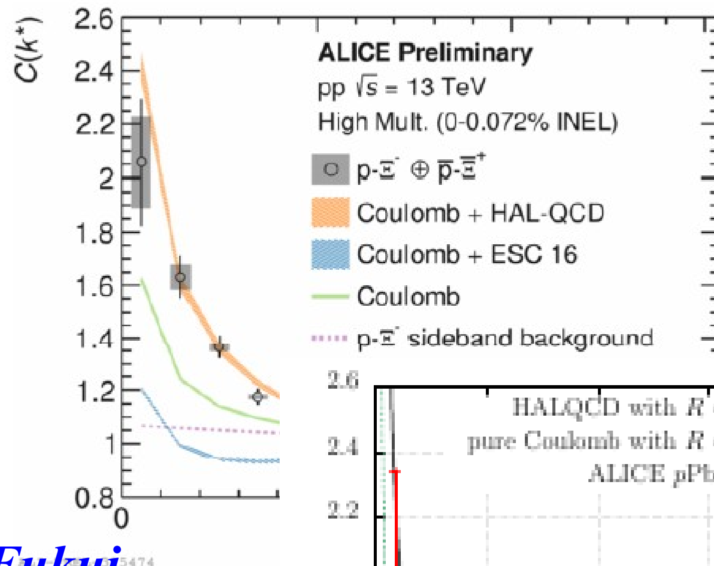
c.f. Hanhart talk (Mon.)

$p\Xi^-$ correlation function

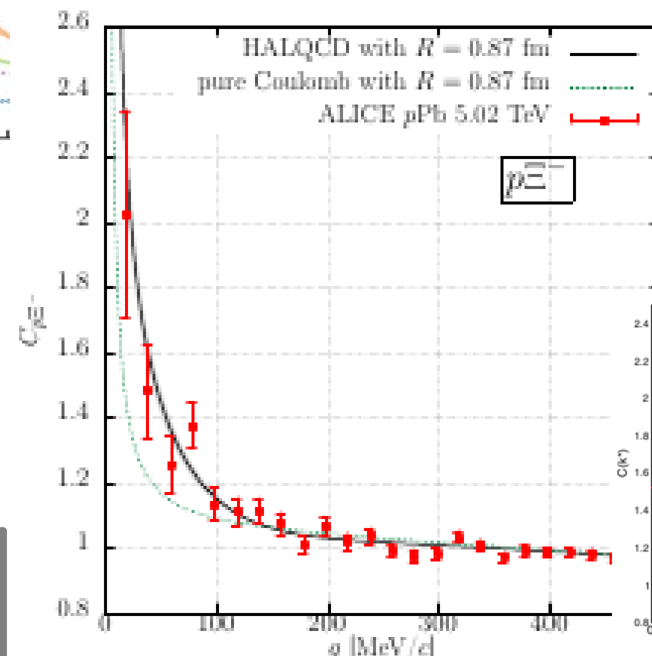


T. Hatsuda, K. Morita, AO, K. Sasaki, NPA967('17)856. (heavier quark mass, I=0 only, w/o CC effects)

J. Haidenbauer, NPA981('19)1. (NLO(600), w/ CC effects, w/o Coulomb) (w/ Coulomb, it will be comparable with data.)

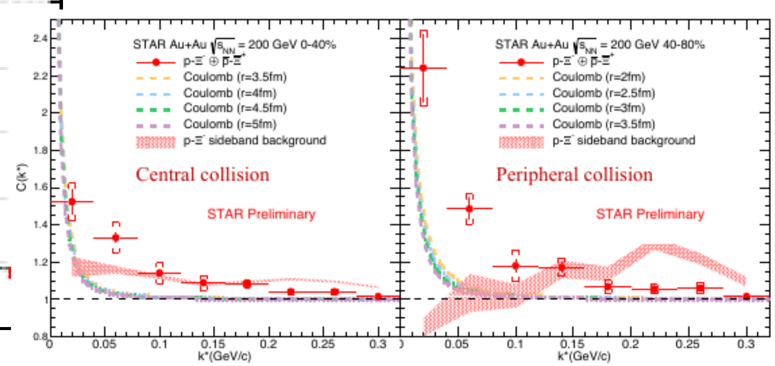


D. L. Mihairov+[ALICE], NPA 1005 ('21)121760 (QM2019). (Nijmegen pot. does not explain the data. w/o CC) Acharya+(ALICE), Nature ('20)



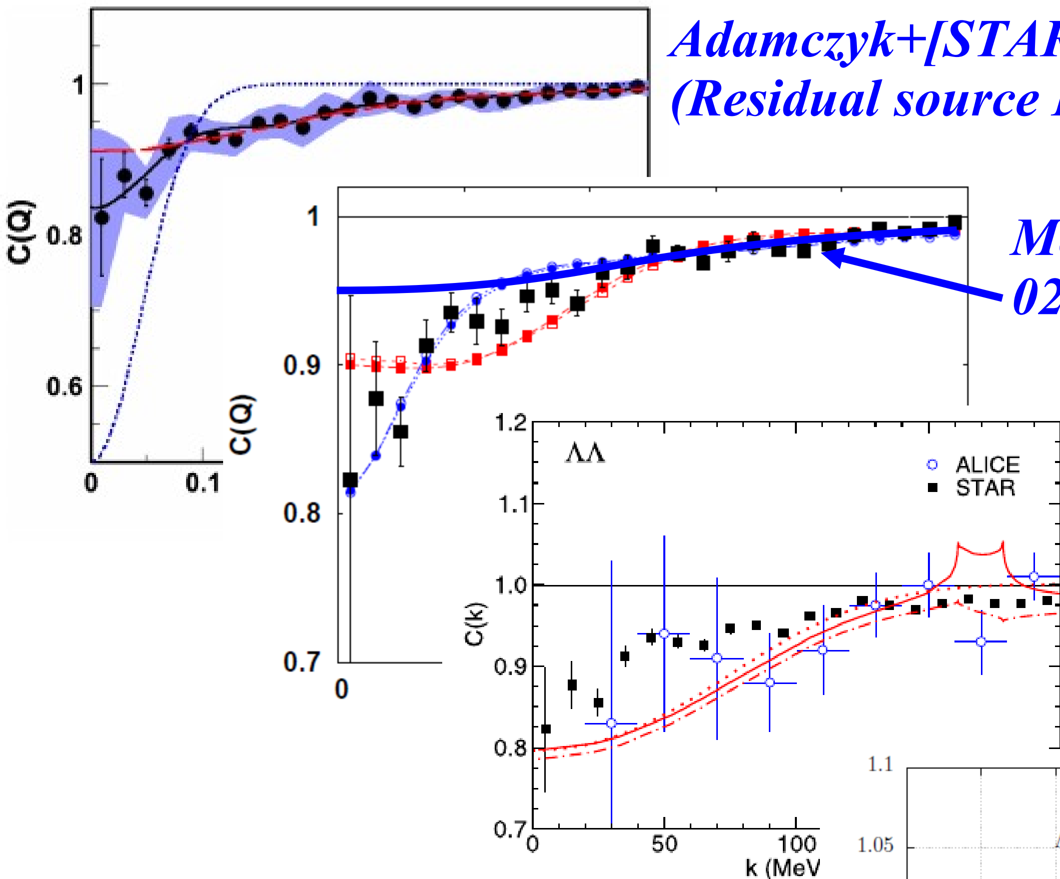
Kamiya, Sasaki, Fukui, Hatsuda, Hyodo, Morita, Ogata, AO (in prep.), w/ Lattice BB pot. at phys. m_q CC effects with ΛΛ.

K. Mi+(STAR, preliminary), Au+Au 200 AGeV, APS2021. (No Dip at larger R)



There is no signal of bound state.

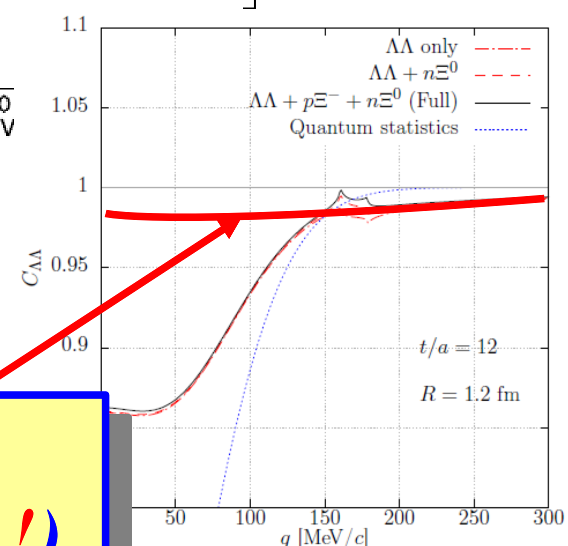
ΛΛ correlation function

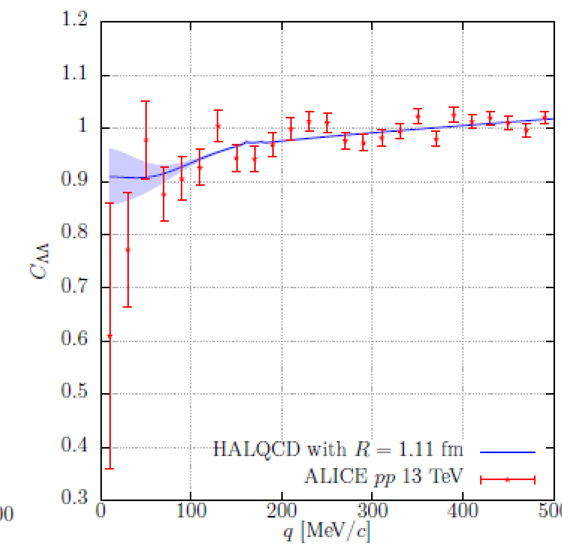


Adamczyk+[STAR], PRL114('15)022301
(Residual source R ~ 0.5 fm was assumed.)

Morita, Furumoto, AO,PRC91('15) 024916. (Res.Source + flow)

J. Haidenbauer, NPA981('19)1. (NLO600)



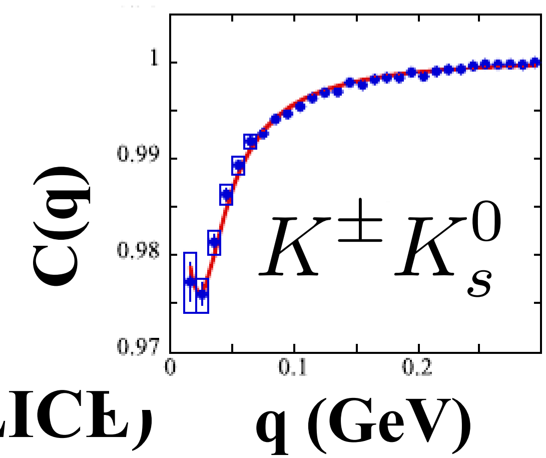


Kamiya+(in prep.).
(CC simulates res. source !)

Recent & Near-Future Correlation Functions

Recent & Near-Future Correlation Functions

- $\overline{p}p$, $p\overline{\Lambda}$ *E.g. A. Kisiel [ALICE], Acta Phys.Polon.Supp. 6 ('13)519*
- $K^{\pm}K^0_s$ *S.Acharya+ [ALICE], PLB774 ('17)64 [1705.04929]*
  → Slightly suppressed at low q
  Tetraquark component of a_0 meson



- **pΛ [2104.04427], pϕ [2105.05578],**
  pΛ, ΛΛ [2105.05190], $p\Sigma^0$ ['20 [1910.14407]] (ALICE)
- **$pD^{\pm}$ (in prog.) Scatt. length is strongly model dependent.**
  → To be discriminated by experiment !

$\overline{D}p$

model	$a_0^{DN(I=0)}$ [fm]	$a_0^{DN(I=1)}$ [fm]	boutnd state (I=0)	bound state (I=1)	
1 [1]	-0.16	-0.26	None	None	*Hofmann+('05)*
2 [2]	0.07	-0.45	None	None	*Haidenbauer+('07)*
3 [3]	-4.38	-0.07	2804	None	*Yamaguchi+('11)*
4 [4]	0.03-0.16	0.20-0.25	None	None	*Fontoura+('13)*

- **deuteron-hadron CF**
  S. Mrówczyński and P. Słoń, Acta Phys.Polon.B51('20)1739 [1904.08320]; F. Etminan, M. M. Firoozabadi, [1908.11484]; J. Haidenbauer, PRC102('20)034001 [2005.05012]; K.Ogata, T.Fukui, Y.Kamiya, AO [2103.00100].

Summary

- **Correlation function is useful to access hadron-hadron interactions as well as to deduce the existence of a bound state.**

pK^-
Chiral CC pot. (examined)
Bound state (favored)

$p\Xi^-$
Lattice QCD CC pot. (examined)
Bound state (disfavored)

$p\Omega$
Lattice QCD pot. J=2 (examined)
Bound state (favored)

	n	p	K^-	K^+	π^-	π^+	Λ	Σ	Ξ^-	Ω^-	D^-	D^+	Ks	+α
n														
p		O	O	O	△	△	O	O	O	O	O	O		
K^-		O	O	O	O	O							O	
K^+		O	O	O	O	O							O	
π^-		△	O	O	O	O								
π^+		△	O	O	O	O								
Λ		O					O							
Σ		O												
Ξ^-		O												
Ω^-		O												
D^-		O												
D^+		O												
Ks			O	O										
+α														

$pD^\pm$
Chamed hadron-nucleon interaction (work in prog.)

$K^\pm K^0_s$
Tetraquark component in a_0 meson

$\Lambda\Lambda$
Scattering pars. (a_0, r_{eff}) (constrained)
Bound state (disfavored)

Summary (cont.)

- Many questions and homeworks
  - In many of previous works, CFs from predicted potentials are compared with data. Is it possible to extract scatt. pars. directly from data ?
  - Source is assumed to be Gaussian and the size is regarded as a parameter in theory papers. Can we use the size determined independently ?
  - How can we calculate three-body CFs ?
    Can we extract 3-body force ?
  - ...
- I'm sorry that I did not refer to numbers.
  Please refer to the papers.

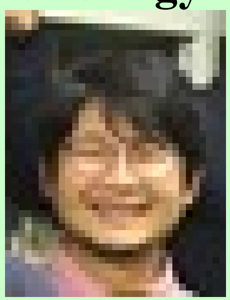
Thank you for your attention !

Thank you for your attention !

Coauthors of arXiv:1908.05414 (pΩ, $\Omega\Omega$) and arXiv:1911.01041 (pK^-), and next paper on pΞ^- , Y. Kamiya, K. Sasaki, T. Fukui, T. Hatsuda, T. Hyodo, K. Morita, K. Ogata, AO, in prep.

K. Morita

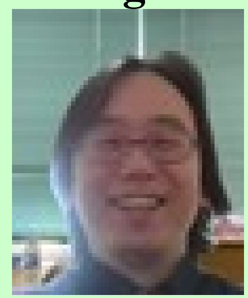
S. Gongyo



T. Hatsuda

T. Hyodo

K. Ogata



T. Fukui

(J. Haidenbauer)

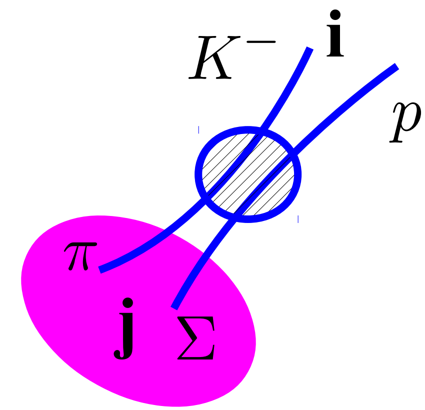
K.Sasaki **Y. Kamiya** **ALICE**

Note on Correlation Function with Coupled Channels

- **Correlation function in the i-th channel**

$$C_i(\boldsymbol{q}) = \sum_\beta \int d\boldsymbol{r}\, \omega_j S_j(\boldsymbol{r}) \left|\psi^{(-)}_{ji}(\boldsymbol{r},\boldsymbol{q})\right|^2$$

Source fn.



- **Asymptotic wave function (s-wave, w/o Coulomb)**
  K. Miyahara, T. Hyodo, W. Weise, PRC98('18)025201 [1804.08269].

$$\psi^{(+)}_{ji}(r;q) \to \frac{-1}{2iq_i}\left[\delta_{ji}\frac{e^{-iq_jr}}{r} - \sqrt{\frac{v_i}{v_j}}S_{ji}\frac{e^{iq_jr}}{r}\right] \quad (v_i = q_i/\mu_i)$$

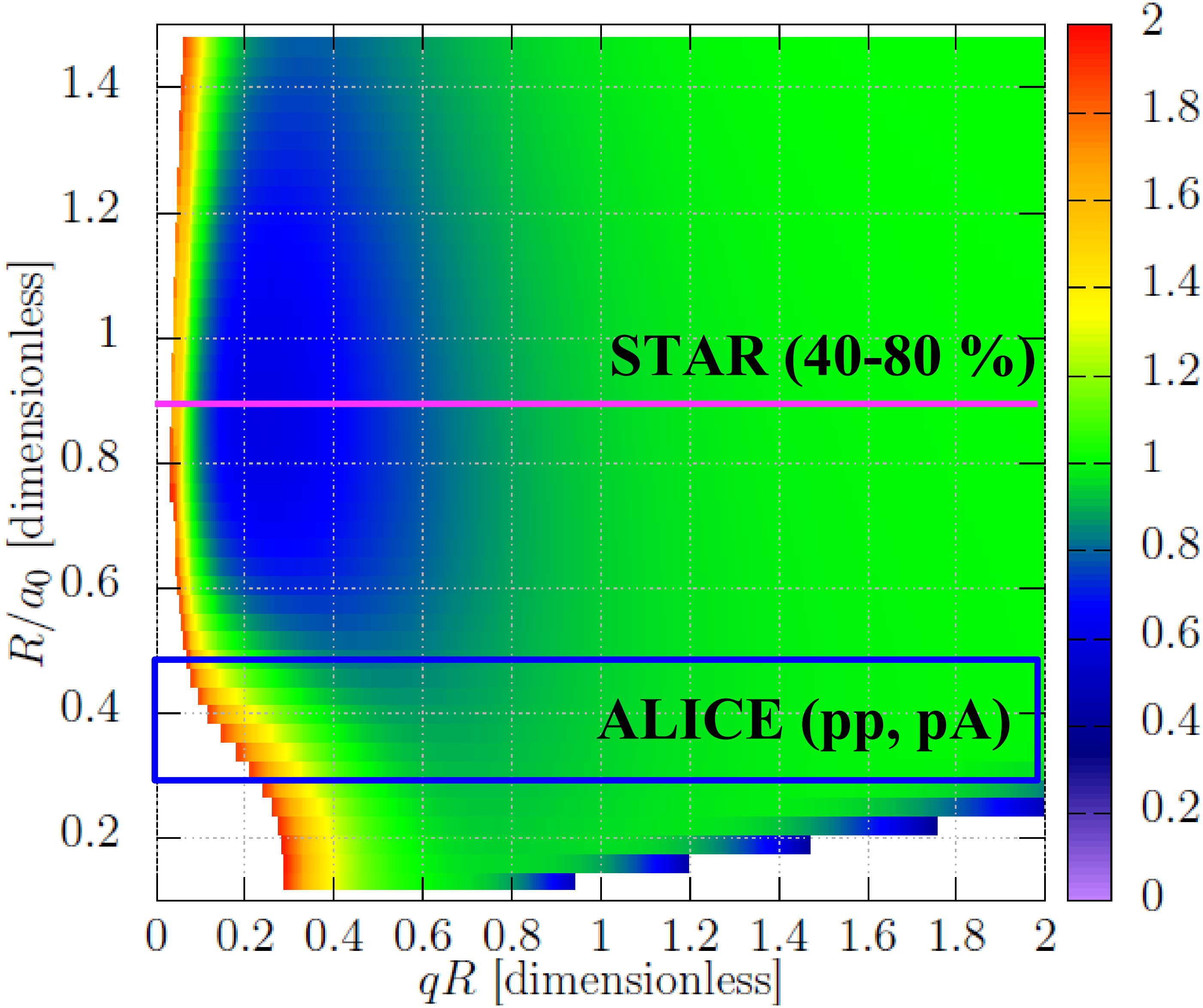
$$\psi^{(-)}_{ji}(r;q) = \frac{1}{q_i}\sum_n \psi^{(+)}_{jn}(r;q)S^\dagger_{ni}q_n\sqrt{\frac{v_i}{v_n}} \to \frac{1}{2iq_i}\left[\delta_{ji}\frac{e^{iq_jr}}{r} - \sqrt{\frac{v_i}{v_j}}S^\dagger_{ji}\frac{e^{-iq_jr}}{r}\right]$$

  - **No incoming w.f. for j≠i in $\psi^{(+)}$**
  - **No outgoing w.f. for j≠i in $\psi^{(-)}$**

- **Correlation function (spherical source)**

$$C_i(\boldsymbol{q}) = 1 - \underbrace{\int d\boldsymbol{r} S_i(\boldsymbol{r})|j_0(qr)|^2}_{\ell \geq 1} + \sum_\beta \int d\boldsymbol{r}\, \omega_j S_j(\boldsymbol{r}) \left|\psi^{(-)}_{ji}(r;q)\right|^2$$

Correlation Function with Gaussian source



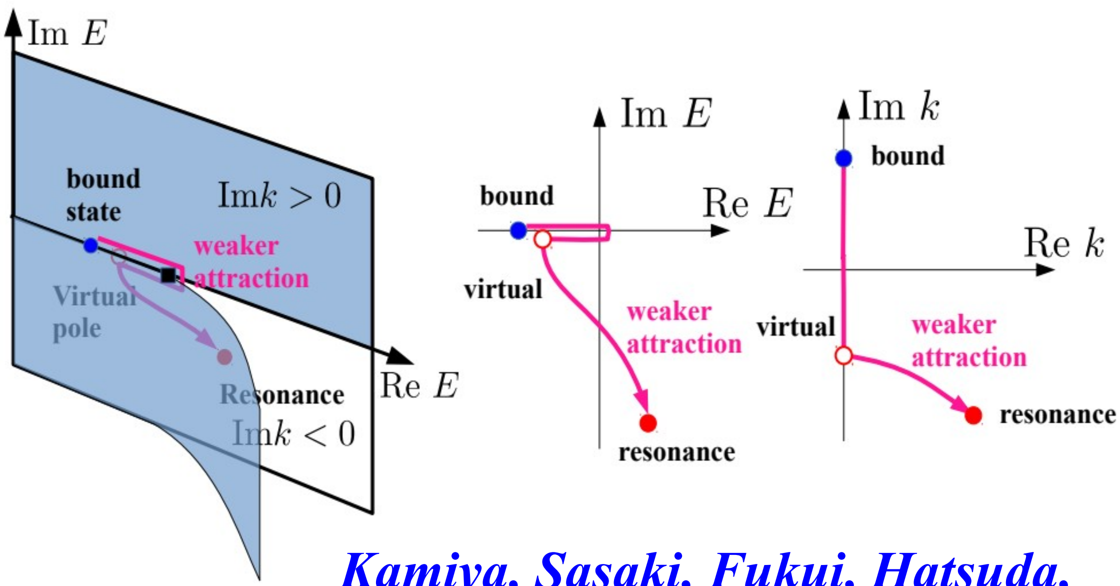
NΩ potential (J=2, HAL QCD, a_0=3.4 fm) + Coulomb

Fate of H dibaryon state ~ Virtual Pole ?

- **Recent HAL QCD results at almost physical quark mass**
  - **There is no bound state in NΞ-ΛΛ system (except for Ξ^- atom), but there is a virtual pole around the NΞ threshold (3.93 MeV below $n\Xi^0$ threshold) on the irrelevant Riemann sheet, (+, –, +) [channels = 1(ΛΛ), 2($n\Xi^0$), 3($p\Xi^-$)]**
  - **Wave function in $n\Xi^0$ channel diverges while the Re(energy) is lower than the threshold → Virtual pole**

$$u_i(r) \propto \exp(iq_i r) = \exp(i\mathrm{Re}(q_i) r) \exp(-\mathrm{Im}(q_i) r)$$

  - **If it appears in the (–,+,+) Riemann sheet, it is a ΛΛ resonance (a NΞ bound state).**



Kamiya, Sasaki, Fukui, Hatsuda, Hyodo, Morita, Ogata, AO, in prep.

Scattering Length

- **pΩ (a_0 in nuclear physics convention)**
  K. Morita, S. Gongyo, T. Hatsuda, T. Hyodo, Y. Kamiya, AO, PRC101('20)015201 [1908.05414]

TABLE III. S-wave scattering length a_0, effective range r_{eff}, and binding energy of the $p\Omega$ pair with the lattice QCD potential for different t/a and the Coulomb attraction.

t/a	a_0 [fm]	r_{eff} [fm]	E_B [MeV]
11	3.45	1.33	2.15
12	3.38	1.31	2.27
13	3.49	1.31	2.08
14	3.40	1.33	2.24

- **K^-N (a_0 in high-energy physics convention)**
  Y. Ikeda, T. Hyodo, W. Weise, NPA881('12) 98 [1201.6549]

$$a(K^-p) = -0.93 + i\,0.82 \text{ fm} \quad (\text{TW}),$$
$$a(K^-p) = -0.94 + i\,0.85 \text{ fm} \quad (\text{TWB})$$
$$a(K^-p) = -0.70 + i\,0.89 \text{ fm} \quad (\text{NLO})$$

$$a(K^-n) = 0.29 + i\,0.76 \text{ fm} \quad (\text{TW})$$
$$a(K^-n) = 0.27 + i\,0.74 \text{ fm} \quad (\text{TWB})$$
$$a(K^-n) = 0.57 + i\,0.73 \text{ fm} \quad (\text{NLO})$$